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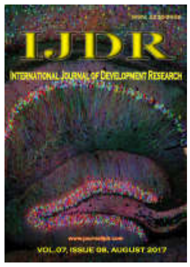

ORIGINAL RESEARCH ARTICLE

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MORE ON WEAKLY FUZZY Δ-SEMI PREIRRESOLUTE MAPPINGS

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ABSTRACT

The aim of this paper is to introduce and investigate the concept of a new class of mappings, called - fuzzy δ - semi preirresolute mappings in fuzzy topological space. Also in this paper some more results on weakly fuzzy δ-semi preirresolute mappings and fuzzy δ-semi preseparation axioms would be investigated in the light of the concepts already introduced on weakly fuzzy δ-semi preirresolute mappings and fuzzy δ-semi preseparation axioms by Mukherjee & Dhar (Mukherjee, 2010)(The Journal of Fuzzy Mathematics, 18(1) 2010, 209-216) in fuzzy topological space.. Also the aim of this paper is to introduce the concept of new fuzzy spaces, named as fuzzy δ-semi preregular space and fuzzy semi δ - preregular space.

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INTRODUCTION

The concept of fuzzy preirresolute mappings was introduced by Park and Park (1994). Again the concept of fuzzy semi preirresolute functions were introduced in fuzzy topological space (fts, for short) by Bhaumik and Mukherjee (1999). Recently Mukherjee and Sarkar (2006) introduced the concept of fuzzy δ-semi irresolute functions in fts. In section 2 of this paper, the concept of a new class of mappings, called - fuzzy δ-semi preirresolute mappings would be introduced and studied in fts. Also Mukherjee and Dhar (2010) introduced the concepts of weakly fuzzy δ-semi preirresolute mappings and fuzzy δ-semi preseparation axioms in fts. In the light of the concept introduced in (2010), some more results on weakly fuzzy δ-semi preirresolute mappings and fuzzy δ-semi preseparation axioms are to be investigated in section 4. In section 5, the concept of a new fuzzy space, named as fuzzy δ -semi preregular space is to be introduced and studied. In section 6, the concept of a new fuzzy space, named as fuzzy semi δ - semi preregular space is to be introduced and studied.Throughout this paper, (X, τ) or simply X will mean a fts due to Chang (1968) and clA, intA, δclA, δintA will denote respectively the closure, interior, δ - closure, δ - interior for a fuzzy subset A of X.

Preliminaries

In this section, some of the known results and definitions are to be mentioned for ready references.

Definition 2.1. A fuzzy subset A in a fts X is called

a) (Ming, 1980) quasi-coincident (q-coincident, for short) with a fuzzy subset B, denoted by AqB, iff ∃ x∈X such that A(x) + B(x) >1,
b) (Ming, 1980) q-coincident with a fuzzy point x_p (where x is the support, p is the value of the point & 0<p≤1) iff p + A(x) >1,
c) (Ming, 1980) q-neighbourhood (q-nbd, for short) of fuzzy point x_p iff there exists a fuzzy open set B such that x_pqB≤A,
d) (Azad, 1981) fuzzy regular open if A= int(cl(A)),
e) (Azad, 1981) fuzzy semiopen if A ≤ clint(A),
f) (Bin Shahana, 1991) fuzzy preopen if A ≤ int(cl(A)),
g) (14) fuzzy semi preopen if A≤cl(int(clA)),
h) (Ganguly, 1988) fuzzy δ-closed iff A=δclA. The complement of fuzzy δ-closed set is called fuzzy δ-open,
i) (Caldas, 1968) fuzzy δ-preopen if A≤int(δclA),
j) (Mukherjee, 2006) fuzzy δ-semi open if A≤cl(δintA),
k) (Thakur et al., 2004) fuzzy δ-semi preopen A≤δcl(intδclA).

Definition 2.2. (Chaudhuri, 1993**)** A fuzzy set A of a fuzzy topological space (X, τ) is said to be RQ-neighbourhood (briefly, RQ-nbd) of a fuzzy point x_p iff there is a fuzzy regular open set B of X such that x_pqB≤A.

Definition 2.3. (Ganguly, 1988) A fuzzy point x_p is said to be fuzzy δ-cluster point of a fuzzy subset A of a fuzzy topological space (X, τ) iff each *RQ-nnbd* of x_p is quasi-coincident with A. The set of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A and is denoted by δclA.

Definition 2.4. Let f : (X, τ) → (Y, σ) be a mapping from a fts (X, τ) to another fts (Y, σ). Then f is called

a) (Mukherjee, 1989) fuzzy irresolute if $f^{-1}(A)$ is a fuzzy semiopen subset in X for each fuzzy semiopen subset A in Y,
b) (Park, 1994) fuzzy preirresolute if $f^{-1}(A)$ is a fuzzy preopen subset in X for each fuzzy preopen subset A in Y,
c) (Bhaumik, 1999) fuzzy semi preirresolute if $f^{-1}(A)$ is a fuzzy semi preopen subset of X for each fuzzy semi preopen subset A in Y,
d) (Mukherjee, 2006) fuzzy δ-semi irresolute if $f^{-1}(A)$ is a fuzzy δ-semiopen subset of X for each fuzzy δ-semiopen subset A in Y.

Definition 2.5. (Mukherjee, 2010) A fuzzy set A of a fts X is called fuzzy δ-semi preneighbourhood (resp. q-neighbourhood) of a fuzzy point x_p if there exists a fuzzy δ-semi preopen set U such that x_p∈U≤A (resp. x_pqU≤A).

Definition 2.6. (Mukherjee, 2010) A fts X is called fuzzy δ-semi pre T_0 iff for every pair of distinct fuzzy points x_p and y_t, the following conditions are satisfied:

when x≠y

either x_p has a fuzzy δ-semi preneighbourhood U such that

U $\not{q}$ y_t or y_t has a fuzzy δ-semi preneighbourhood V such that

V $\not{q}$ x_p,

when x=y

and p<t (say), there is a fuzzy δ-semi pre q-neighbourhood V of y_t such that V $\not{q}$ x_p.

Definition 2.7. (Mukherjee, 2010). A fts X is called fuzzy δ-semi pre T_1 iff for every pair of distinct fuzzy points x_p and y_t, the following conditions are satisfied**:**

when x≠y, x_p

has a fuzzy δ-semi preneighbourhood U such that U $\not{q}$ y_t and y_t

has a fuzzy δ-semi preneighbourhood V such that V $\not{q}$ x_p,

when x=y

and p<t (say), there exists a fuzzy δ-semi pre q-neighbourhood V of y_t

such that V $\not{q}$ x_p.

Definition 2.8. (Mukherjee, 2010). A fts X is called fuzzy δ-semi pre T_2 iff for every pair of distinct fuzzy points x_p and y_t, the following conditions are satisfied:

- when x≠y, x_p and y_t has fuzzy δ-semi preneighbourhoods U and V respectively such that U $\not{q}$ V,
- when x=y and p<t (say), x_p has a fuzzy δ-semi preneighbourhood U and y_t has a
- fuzzy δ-semi pre q-neighbourhood V such that U $\not{q}$ V.

Lemma 2.9. (Azad, 1981)Let f : X→Y be a mapping and {B_i}be a family of fuzzy sets of Y, then

f^{-1}(B_i) = f^{-1}(B_i)

and

f^{-1}(B_i) = f^{-1}(B_i).

Lemma 2.10. (Azad, 1981)For mappings f_i X_i→Y_i and fuzzy sets B_i of Y_i, i = 1, 2; we have

$(f_1 \times f_2)^{-1}(B_1 \times B_2) = f^{-1}(B_1) \times f^{-1}(B_2)$.

Lemma 2.11. (Azad, 1981)Let g: X→X×Y be the graph of a mapping f : X→Y. Then if A is a fuzzy set of X and B is a fuzzy set of Y, g^{-1}(A×B)=A f^{-1}(B).

Fuzzy δ - semi preirresolute mappings

In this section, a new class of mappings, called - fuzzy δ - semi preirresolute mappings are to be defined with the help of fuzzy δ - semi preopen sets. Also some of their basic properties and characterization theorems are to be investigated in fuzzy topological spaces.

Definition 3.1. A mapping f : (X, τ) → (Y, σ) from a fuzzy topological space (X, τ) to another fuzzy topological space (Y, σ) is called fuzzy δ - semi preirresolute if $f^{-1}(B) \in \delta spo(X)$ for each $B \in \delta spo(Y)$. Here δspo(X) and δspo(Y) denote fuzzy. δ - semi preopen sets on X and Y respectively.

Theorem 3.2. Let f : (X, τ) → (Y, σ) from a fuzzy topological space (X, τ) to another fuzzy topological space (Y, σ). Then the following statements are equivalent:

- f is fuzzy δ - semi preirresolute.
- For every fuzzy set $B \in \delta spc(Y)$, $f^{-1}(B) \in \delta spc(X)$.
- For every fuzzy point x_p in X and every fuzzy set $B \in \delta spo(Y)$ such that $f(x_p) \in B$, there is a fuzzy set $A \in \delta spo(X)$ such that $x_p \in A$ and $f(A) \le B$.
- For every fuzzy point x_p in X and every fuzzy nbd $A \in \xi(f(x_p))$, there is a fuzzy nbd $f^{-1}(A) \in \xi(x_p)$.
- For every fuzzy point x_p in X and every fuzzy nbd $A \in \xi(f(x_p))$, there is a fuzzy nbd $B \in \xi(x_p)$ of x_p such that $f(B) \le A$.

Proof:

(a) ⇒ (b) : Let $B \in \delta spc(Y)$, then $(1_Y - B) \in \delta spo(Y)$. By (a), $f^{-1}(1_Y - B) = (1_Y - f^{-1}(B)) \in \delta spo(X)$, i.e., $f^{-1}(B) \in \delta spc(X)$.
(b) ⇒ (a) : Let B be any fuzzy δ - semi preopen set in Y, i.e., $B \in \delta spo(Y)$. Then $1_Y - B$ is a fuzzy δ - semi preclosed set in Y, i.e., $(1_Y - B) \in \delta spc(Y)$. Now, by (b), $f^{-1}(1_Y - B) = (1_Y - f^{-1}(B))$ is fuzzy δ - semi preclosed in X. Hence $f^{-1}(B)$ is fuzzy δ - semi preopen set in X, i.e., $f^{-1}(B) \in \delta spo(X)$. Hence f is fuzzy δ - semi preirresolute.

(a) ⇒ (c): Let x_p be a fuzzy point of X and B be a fuzzy δ - semi preopen set in Y, i.e., $B \in \delta spo(Y)$ with $f(x_p) \in B$. Put $A = f^{-1}(B)$. Then by (a), $A \in \delta spo(X)$ such that $x_p \in A$ and $f(A) \le B$.

(c) ⇒ (a) : Let B be any fuzzy δ - semi preopen set in Y, i.e., $B \in \delta spo(Y)$ and a fuzzy point $x_p \in f^{-1}(B)$. Then $f(x_p) \in B$. Now by (c), there is a fuzzy set $A \in \delta spo(X)$ such that $x_p \in A$ and $f(A) \le B$. Then $x_p \in A \le f^{-1}(B)$. Hence by theorem – A fuzzy set $A \in \delta spo(X)$ if and only if for every fuzzy point $x_p \in A$, there exists a fuzzy set $B \in \delta spo(X)$ such that $x_p\, q\, B \le A$, $f^{-1}(B) \in \delta spo(X)$. Thus f is fuzzy δ - semi preirresolute.

(a) ⇒ (d) : Let x_p be a fuzzy point of X and A be fuzzy δ - semi pre nbd of $f(x_p)$, i.e., $A \in \xi(f(x_p))$. Then there is a fuzzy δ - semi preopen set B such that $f(x_p) \in B \le A$. Now by (a), $f^{-1}(B) \in \delta spo(X)$ and $x_p \in f^{-1}(B) \le f^{-1}(A)$. Thus $f^{-1}(A)$ is a fuzzy δ - semi pre nbd of x_p in X, i.e., $f^{-1}(A) \in \xi(x_p)$.
(d) ⇒ (e) : Let x_p be a fuzzy point of X and A be a fuzzy δ - semi pre nbd of $f(x_p)$, i.e., $A \in \xi(f(x_p))$. Then $B = f^{-1}(A)$ is a fuzzy δ - semi pre nbd of x_p, i.e., $f^{-1}(A) \in \xi(x_p)$ and $f(B) = f(f^{-1}(A)) \le A$.

(e) ⇒ (c) : Let x_p be a fuzzy point of X and B be a fuzzy δ - semi preopen set in Y such that $f(x_p) \in B$. Then B is a fuzzy δ - semi pre nbd of $f(x_p)$, i.e., $B \in f(\xi(x_p))$. So by (e), there is a fuzzy δ - semi pre nbd A of x_p in X such that $x_p \in A$ and $f(A) \le B$. Hence there is a fuzzy set $C \in \delta spo(X)$ such that $x_p \in C \le A$ and so $f(C) \le f(A) \le B$.

Weakly fuzzy δ-semi preirresolute mappings

Mukherjee and Dhar (Mukherjee, 2010)defined weakly fuzzy δ-semi preirresolute mappings. Some of their basic properties & characterization theorems were also investigated by them. Fuzzy δ - semi preseparation axioms were also introduced by Mukherjee and Dhar (2010). In this section some more results on weakly fuzzy δ - semi preirresolute mappings and some more fundamental properties on fuzzy δ - semi preseparation axioms are to be investigated in the light of the concepts as introduced by Mukherjee and Dhar (2010).

Definition 4.1. (Mukherjee, 2010) Let f : (X, τ)→(Y, σ) be a mapping from a fts (X, τ) to another fts (Y, σ). Then f is called weakly fuzzy δ-semi preirresolute if $f^{1}(A)$ is a fuzzy δ-semi preopen subset of X for each fuzzy δ - preopen subset A in Y.

Example 4.2. (Mukherjee, 2010) Let V_1, V_2, V_3, V_4 be fuzzy sets on X = {a, b, c} defined as

$V_1(a) = 0.8, V_1(b) = 0.7, V_1(c) = 0.9,$
$V_2(a) = 0.5, V_2(b) = 0.3, V_2(c) = 0.6,$
$V_3(a) = 0.3, V_3(b) = 0.4, V_3(c) = 0.3,$
$V_4(a) = 0.2, V_4(b) = 0.6, V_4(c) = 0.2.$
Let $\tau = \{1_X, 0_X, V_1, V_2, V_3, V_2 \vee V_3, V_2 \wedge V_3\}$ be a fuzzy topology on X.

Then V_4 is fuzzy δ - semi preopen on X.

Define f : (X, τ) → (X, τ) by f(x) = x. Then f is fuzzy δ - semi preirresolute, V_4 is not fuzzy δ - preopen. So, f is not weakly fuzzy δ - semi preirresolute.

Example 4.3. (Mukherjee, 2010)Let V_1, V_2, V_3 and V_4 be fuzzy sets on X = {a, b, c} as defined in example 3(A).3.2. Let $\tau_1 = \{1_X, 0_X, V_1, V_2, V_3, V_2 \wedge V_3, V_2 \vee V_3\}$ and $\tau_2 = \{1_X, 0_X, V_4\}$ be fuzzy topologies on X and f : (X, τ₁) → (X, τ₂) be mapping defined as follows :

f(a) = a, f(b) = b & f(c) = c

Then f is weakly fuzzy δ - semi preirresolute.

Theorem 4.4. If f:X→Y is a weakly fuzzy δ-semi preirreselote mapping and g:Y→Z is a fuzzy δ-preirreselote mapping, then gof:X→Z is a weakly fuzzy δ-semi preirreselote mapping.

Proof. Let C be an arbitrary fuzzy δ-preopen set of Z. As g is fuzzy δ-preirreselote, so $g^{-1}(C)$ is fuzzy δ-preopen set of Y. Since $g^{-1}(C)$ is fuzzy δ-preopen set of Y and f is weakly fuzzy δ-semi preirreselote, so $f^{1}(g^{-1}(C))$ is fuzzy δ-semi preopen set of X. But $f^{1}(g^{-1}(C)) = (gof)^{-1}(C)$. Therefore for each fuzzy δ-preopen set of Z, $(gof)^{-1}(C)$ is fuzzy δ-semi preopen set of X. This shows that gof:X→Z is a weakly fuzzy δ-semi preirreselote mapping.

Theorem 4.5. Let X and Y be fts such that X is product related to Y and f: (X, τ)→(Y, σ) be a mapping. Then, if the graph mapping g:(X, τ)→(X× Y, θ) of f defined by g(x)=(x, f(x)) is weakly fuzzy δ-semi preirresolute then f is also weakly fuzzy δ-semi preirresolute.

Proof: Let g be weakly fuzzy δ-semi preirresolute and B be any fuzzy set in Y. Then by Lemma 2.11., $f^{1}(B) = 1_x \quad f^{1}(B) = g^{-1}(1_x \times B)$. Now, if B is fuzzy open in Y, then $1_x \times B$ is fuzzy open in X × Y. Again, $g^{-1}(1_x \times B)$ is fuzzy δ-semi preopen as g is weakly fuzzy δ-semi preirresolute. Consequently, $f^{1}(B)$ is fuzzy δ-semi preopen. Hence f is weakly fuzzy δ-semi preirresolute.

Theorem 4.6. Let (X_1, τ), (X_2, ω), (Y_1, η) and (Y_2, σ) be fts such that X_1 is product related to X_2. Then, the product $f_1 \times f_2$: $(X_1 \times X_2, \theta) \rightarrow (Y_1 \times Y_2, \rho)$, where θ (resp. ρ) is the fuzzy product topology generated by τ and ω (resp. η and σ) of weakly fuzzy δ-semi preirresolute mappings f_1: $(X_1, \tau) \rightarrow (Y_1, \eta)$ and f_2: $(X_2, \omega) \rightarrow (Y_2, \sigma)$ is weakly fuzzy δ-semi preirresolute.

Proof. Let A= $_{m, n} (A_m \times B_n)$, where A_m, B_n are fuzzy δ-preopen sets in Y_1 and Y_2 respectively. Then A is a fuzzy δ-preopen set of $Y_1 \times Y_2$. By Lemmas 2.9. and 2.10., we have

$(f_1 \times f_2)^{-1}(A) = (f_1 \times f_2)^{-1}(_{m, n} (A_m \times B_n))$

$= _{m, n} ((f_1 \times f_2)^{-1} (A_m \times B_n))$

$= _{m, n} ((f_1^{-1}(A_m) \times f_2^{-1}(B_n))$

Since f_1 and f_2 are weakly fuzzy δ-semi preirresolute, $f_1^{-1}(A_m)$'s are fuzzy δ-semi preopen sets of X_1 and $f_2^{-1}(B_n)$'s are fuzzy δ-semi preopen sets of X_2. So $(f_1^{-1}(A_m) \times f_2^{-1}(B_n))$'s are fuzzy δ-semi preopen sets of $X_1 \times X_2$. As any union of fuzzy δ-semi preopen sets of a fts X is fuzzy δ-semi preopen sets of X, it follows that $(f_1 \times f_2)^{-1}(A)$ is a fuzzy δ-semi preopen set in $X_1 \times X_2$ which implies that $f_1 \times f_2$ is weakly fuzzy δ-semi preirresolute.

Fuzzy δ-semi preseparation axioms

Fuzzy δ-semi preseparation axioms were introduced by Mukherjee and Dhar (2010). In this section some more fundamental properties of this new class of separation axioms are to be investigated.

Theorem 5.1. Let f:X→Y be injective and fuzzy δ-semi precontinuous. If Y is fuzzy pre-T_i then X is fuzzy δ-semi pre T_i for i=0, 1, 2.

Proof.

We give a proof for i=0. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x) ≠f(y). By the fuzzy pre-T_0 property of Y, either there exists a fuzzy preneighbourhood U of $(f(x))_p$ such that U is not q-coincident with $(f(y))_t$ or there exists a fuzzy pre neighbourhood V of $(f(y))_t$ such that V is not q-coincident with $(f(x))_p$. Since f is fuzzy δ-semi precontinuous $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi preneighbourhoods of x_p and y_t respectively such that x_p is not q-coincident with $f^{-1}(V)$ or y_t is not q-coincident with $f^{-1}(U)$. When x=y and p<t (say), then f(x)=f(y). Since Y is fuzzy pre T_0, there exists a fuzzy pre q-nbd Vof $(f(y))_t$ such that $(f(x))_p$ is not q-coincident with V. Since f is fuzzy δ-semi precontinuous, so $f^{-1}(V)$ is a fuzzy δ-semi pre q-nbd of y_t such that x_p is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre T_0.

We give a proof for i=1. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x) ≠f(y). By the fuzzy pre-T_1 property of Y, there exist fuzzy preneighbourhoods U & V of $(f(x))_p$ and $(f(y))_t$ respectively such that $(f(x))_p$ is not q-coincident with V and $(f(y))_t$ is not q-coincident with U. Since f is fuzzy δ-semi precontinuous $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi preneighbourhoods of x_p and y_t respectively such that x_p is not q-coincident with $f^{-1}(V)$ and y_t is not q-coincidnt with $f^{-1}(U)$. When x=y and p<t (say), then f(x)=f(y). Since Y is fuzzy pre T_1, there exists a fuzzy pre q-nbd V of $(f(y))_t$ such that $(f(x))_p$ is not q-coincident with V. Since f is fuzzy δ-semi precontinuous, so $f^{-1}(V)$ is a fuzzy δ-semi pre q-nbd of y_t such that x_p is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre T_1. We give a proof for i=2. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x) ≠f(y). By the fuzzy pre-T_2 property of Y, there exist fuzzy preneighbourhoods U & V of $(f(x))_p$ and $(f(y))_t$ respectively such that U is not q-coincident with V. Since f is fuzzy δ-semi precontinuous $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi preneighbourhoods of x_p and y_t respectively such that $f^{-1}(U)$ is not q-coincident with $f^{-1}(V)$. When x=y and p<t (say), then f(x)=f(y). Since Y is fuzzy pre-T_2, there exists a fuzzy prenbd U of $(f(x))_p$ and there exists a fuzzy pre q-nbd V of $(f(y))_t$ such that U is not q-coincident with V. Since f is fuzzy δ-semi precontinuous, so $f^{-1}(U)$ is a fuzzy δ-semi prenbd of x_p and $f^{-1}(V)$ is a fuzzy δ-semi pre q-nbd of y_t such that $f^{-1}(U)$ is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre- T_2.

Theorem 5.2. Let f:X→Y be a one-to-one fuzzy δ-semi preirresolute mapping. If Y is fuzzy δ-semi pre-T_i then so is X, for i=0, 1, 2.

Proof. We give a proof for i=0. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x) ≠f(y). By the fuzzy δ-semi pre-T_0 property of Y, either there exists a fuzzy δ-semi preneighbourhood U of $(f(x))_p$ such that U is not q-coincident with $(f(y))_t$ or there exists a fuzzy δ-semi preneighbourhood V of $(f(y))_t$ such that V is not q-coincident with $(f(x))_p$. Since f is fuzzy δ-semi preirresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi pre neighbourhoods of x_p and y_t respectively such that x_p is not q-coincident with $f^{-1}(V)$ or y_t is not q-coincidnt with $f^{-1}(U)$. When x=y and p<t (say), then f(x)=f(y). Since Y is fuzzy δ-semi pre-T_0, there exists a fuzzy δ-semi pre q-neighbourhood V of $(f(y))_t$ such that $(f(x))_p$ is not q-coincident with V. Since f is fuzzy δ-semi preirresolute, so $f^{-1}(V)$ is a fuzzy δ-semi pre q-neighbourhood of y_t such that x_p is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre T_0.

We give a proof for i=1. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x)≠f(y). By the fuzzy δ-semi pre-T_1 property of Y, there exist fuzzy δ-semi pre neighbourhoods U & V of $(f(x))_p$ and $(f(y))_t$ respectively such that $(f(x))_p$ is not q-coincident with V and $(f(y))_t$ is not q-coincident with U. Since f is fuzzy δ-semi preirresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi preneighbourhoods of x_p and y_t respectively such that x_p is not q-coincident with $f^{-1}(V)$ and y_t is not q-coincidnt with $f^{-1}(U)$. When x=y and p<t (say), then f(x)=f(y). Since Y is fuzzy δ-semi pre-T_1, there exists a fuzzy δ-semi pre q-neighbourhood V of $(f(y))_t$ such that $(f(x))_p$ is not q-coincident with V. Since f is fuzzy δ-semi preirresolute, so $f^{-1}(V)$ is a fuzzy δ-semi pre q-nbd of y_t such that x_p is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre T_1.

We give a proof for i=2. Let x_p and y_t be two distinct fuzzy points in X. When x≠y, we have f(x)≠f(y). By the fuzzy δ-semi pre-T_2 property of Y, there exist fuzzy δ-semi pre neighbourhoods U & V of $(f(x))_p$ and $(f(y))_t$ respectively such that U is not q-coincident with V. Since f is fuzzy δ-semi preirresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy δ-semi pre neighbourhoods of x_p and y_t respectively such that $f^{-1}(U)$ is not q-coincident with $f^{-1}(V)$. When x=y and p<t (say), then

f(x)=f(y). Since Y is fuzzy δ-semi pre-T_2, there exists a fuzzy preneighbourhood U of $(f(x))_p$ and there exists a fuzzy pre q-neighbourhood V of $(f(y))_t$ such that U is not q-coincident with V. Since f is fuzzy δ-semi preirresolute, so $f^{-1}(U)$ is a fuzzy δ-semi preneighbourhood of x_p and $f^{-1}(V)$ is a fuzzy δ-semi pre q-neighbourhood of y_t such that $f^{-1}(U)$ is not q-coincident with $f^{-1}(V)$. Hence X is fuzzy δ-semi pre-T_2.

Fuzzy δ-semi preregular space

In this section a new fuzzy space, called 'fuzzy δ - semi preregular space' is to be introduced and some basic properties related to this space are also to be investigated. The family of fuzzy δ - semi preopen (resp. δ - semi preclosed) sets of a fts X will be denoted by ψ(x) (respectively ψ (x)) and the set of all fuzzy δ - semi preneighbourhoods (respectively δ - semi pre q - neighbourhoods) of x_p will be denoted by $\xi(x_p)$ (respectively $\eta(x_p)$).

Definition 6.1. A fts X is called fuzzy δ-semi preregular iff for each fuzzy point x_p in X and each U ∈ ψ(X) with $U \in \eta(x_p)$, there is a V ∈ ψ(X) and $V \in \eta(x_p)$ such that clV ≤ U.

Lemma 6.2. Let (X, τ) be a fuzzy topological space and two fuzzy sets U, V ∈ δspo(X). If U $\not q$ V, then cl(U) $\not q$ V.

Proof : Let U $\not q$ V. Then U ≤ V^c. Since V^c ∈ δspc(X), cl(U) ≤ V^c. It implies that cl(U) $\not q$ V.

Theorem 6.3. For a fts X, the following statements are equivalent:

a) X is fuzzy δ-semi preregular.
b) For each fuzzy point x_p in X and each B ∈ ψ (X) with $x_p \notin$ B, there is
c) a U ∈ ψ(X) such that $x_p \notin$ clU and B ≤ U.
d) For each fuzzy point x_p in X and each B ∈ ψ (X) with $x_p \notin$ B, there exist U, V ∈
e) ψ(X) such that $U \in \eta(x_p)$, B ≤ V and U $\not q$ V.
f) For any fuzzy set A and any B ∈ ψ (X) with A is not less than equal to B, there are U, V ∈ ψ(X) such that AqU, B ≤ V and U $\not q$ V.
g) For any fuzzy set A and any U∈ψ(X) with AqU, there is a V∈ ψ(X) such that AqV≤ clV ≤ U.

Proof.

(a)⟹(b): Let x_p be a fuzzy point in X and B∈ ψ (X) with $x_p \notin$ B. Then $B^c \in \eta(x_p)$ and B^c ∈ ψ(X). Since X is fuzzy δ-semi preregular, there is a V ∈ ψ(X) and $V \in \eta(x_p)$ such that clV ≤ B^c. Put U = $(clV)^c$. Then U∈ψ(x) and intclV∈$\eta(x_p)$. Hence $x_p \notin (intclV)^c = cl(clV)^c$ = clU and B ≤ $(clV)^c$ = U. **(b)⟹(c):** For any fuzzy point x_p in X and any B∈ψ (X) with $x_p \notin$B, by (b) there is a U∈ ψ(X) such that $x_p \notin$clU and B≤U. Hence $(clU)^c \in \eta(x_p)$ and $(clU)^c$ $\not q$ U, where $(clU)^c$ ∈ ψ(X). Put $(clU)^c$ = V, we obtain U $\not q$ V. Thus (c) is obtained.

(c)⟹(d): Let A be a fuzzy set and B ∈ ψ (X) with A is not less than equal to B. Then there is at least one fuzzy point x_p ∈ A such that $x_p \notin$ B. By (c), there are U, V∈ ψ(X) such that U $\in \eta(x_p)$, B ≤ V and U $\not q$ V. Since x_p ∈ A, we have AqU.

(d)⟹(e): For any fuzzy set A and any U∈ψ(X), AqU implies that A∉U^c, where U^c ∈ ψ (X). By (d), there are V, W∈ψ(X) such that AqV, U^c ≤ W and V $\not q$ W. Then by Lemma 6.2., we get clV $\not q$ W. Hence AqV ≤ clV ≤ W^c ≤ U.

(e)⟹(a): Let x_p be a fuzzy point in X. By (e), for any fuzzy set A and any U ∈ ψ(X) with AqU, there is a V∈ ψ(X) such that AqV ≤ clV ≤ U which implies that X is fuzzy δ-semi pre-regular.

Fuzzy semi δ - preregular space

In this section, the concept of fuzzy semi δ - preregular space is to be introduced and studied with the help of fuzzy semi δ - preopen set and fuzzy semi δ - pre q - nbd. The family of fuzzy semi - δ preopen (respectively semi - δ preclosed) sets of a fts X will be denoted by sδpo(X) (respectively sδpc(X)) and the set of all fuzzy semi - δ - preneighbourhoods (respectively semi - δ pre q - neighbourhoods) of x_p will be denoted by $PN(x_p)$ (respectively $PNq(x_p)$).

Definition 7.1. A fuzzy topological space (X, τ) is called fuzzy semi δ - pre regular if and only if for each fuzzy point x_p in (X, τ) and each fuzzy set U ∈ sδpo(X) with $U \in PNq(x_p)$, there is a fuzzy set V ∈ sδpo(X) and $V \in PNq(x_p)$ such that cl(V) ≤ U.

Lemma 7.2. Let (X, τ) be a fuzzy topological space and two fuzzy sets U, V ∈ sδpo(X). If U $\not q$ V, then cl(U) $\not q$ V.

Proof : Let U $\not q$ V. Then U ≤ V^c. Since V^c ∈ sδpc(X), cl(U) ≤ V^c. It implies that cl(U) $\not q$ V.

Theorem 7.3. For a fuzzy topological space (X, τ), the following conditions are equivalent:

a) X is fuzzy semi δ - preregular.
b) For each fuzzy point x_p in X and each fuzzy set B ∈ sδpc(X) with $x_p \notin$ B, there is a fuzzy set U ∈ sδpo(X) such that $x_p \notin$ cl(U) and B ≤ U.
c) For each fuzzy point x_p in X and each fuzzy set B ∈ sδpc(X) with $x_p \notin$ B, there exist fuzzy sets U, V ∈ sδpo(X) such that $U \in PNq(x_p)$, B ≤ V and U $\not q$ V.
d) For any fuzzy set A and any fuzzy set B ∈ sδpc(X) with A $\not\leq$ B, there are fuzzy sets U, V ∈ sδpo(X) such that A q U, B ≤ V and U $\not q$ V.
e) For any fuzzy set A and any fuzzy set U ∈ sδpo(X) with A q U, there is a fuzzy set V ∈ sδpo(X) such that A q V ≤ cl(V) ≤ U.

Proof:

(a) ⇒ (b) : Let x_p be a fuzzy point in X and a fuzzy set B ∈ sδpc(X) with $x_p \notin$ B. Then $B^c \in PNq(x_p)$ and B^c ∈ sδpo(X).

Since X is fuzzy semi δ - preregular, there is a fuzzy set V ∈ sδpo(X) and V ∈ PNq(x_p) such that cl(V) ≤ B^c. Put U = $(cl(V))^c$. Then a fuzzy set U ∈ sδpo(X) and intcl(V) ∈ PNq(x_p). Hence x_p ∉ $(intcl(V))^c$ = $cl(cl(V))^c$ = cl(U) and B ≤ $(cl(V))^c$ = U.

(b) ⇒ (c) : For any fuzzy point x_p in X and any fuzzy set B ∈ sδpc(X) with x_p ∉ B, by (b) there is a fuzzy set U ∈ sδpo(X) such that x_p ∉ cl(U) and B ≤ U. Hence $(cl(U))^c$ ∈ PNq(x_p) and $(cl(U))^c$ $\not{q}$ U, where $(cl(U))^c$ ∈ sδpo(X). Put $(cl(U))^c$ = V, we obtain U $\not{q}$ V. Thus (c) is obtained.

(c) ⇒ (d) : Let A be a fuzzy set and a fuzzy set B ∈ sδpc(X) with A ≰ B. Then there is at least one fuzzy point x_p ∈ A such that x_p ∉ B. By (c), there are fuzzy sets U, V ∈ sδpo(X) such that U ∈ PNq(x_p), B ≤ V and U $\not{q}$ V. Since x_p ∈ A, we have A q U.

(d) ⇒ (e) : For any fuzzy set A and any fuzzy set U ∈ sδpo(X), A q U implies that A ≰ U^c, where U^c ∈ sδpc(X). By (d), there are two fuzzy sets V, W ∈ sδpo(X) such that A q V, U^c ≤ W and V $\not{q}$ W. Then by lemma 7.2., we get cl(V) $\not{q}$ W. Hence A q V ≤ cl(V) ≤ W^c ≤ U.

(e) ⇒ (a) : Let x_p be a fuzzy point in X. By (e), for any fuzzy set A and any fuzzy
set U ∈ sδpo(X) with A q U, there is a fuzzy set V ∈ sδpo(X) such that A q V ≤ cl(V) ≤ U which implies that X is fuzzy semi δ - preregular.

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