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MATHEMATICAL MODELING OF BLOOD FLOW THROUGH TAPERED STENOSED ARTERY WITH THE SUSPENSION OF NANOPARTICLES USING JEFFREY FLUID MODEL

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ABSTRACT

This research paper presents the mathematical analysis of heat and mass transfer on the blood flow through a tapered stenosed artery with the suspension of nanoparticles. The non-linear equations governing blood flow is modeled in cylindrical co-ordinates and are solved by Homotopy Perturbation Method where the rheology of flowing blood is characterized by the Jeffrey fluid model. Analytical solutions are constructed for the velocity profile, temperature profile, concentration profile and flux profile by solving flow governing non-linear coupled equations. Homotopy Perturbation technique is used to calculate velocity profile, temperature profile, concentration profile and flux profile for different values of thermophoresis and Brownian motion parameter. MATLAB computational programming is used to find the computational results. The computational results are presented graphically at the end of the paper. The significance of the present model over the existing models has been pointed out by comparing the result with other theories both analytically and numerically as to validate the applicability of this mathematical model where the blood is considered as Jeffrey fluid model.

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INTRODUCTION

Nowadays the investigation of blood flow analysis in a constricted stenosed artery is very important in the medical domain because of the fact that many of the diseases such as heart attacks and strokes are related to blood flow and also the physical characteristics of the vessel wall (Sheikholeslami *et al*., 2015 Mekheimen *et al*., 2011). At present the leading causes of the death in the world are due to heart diseases such as atherosclerosis. Atherosclerosis occurs where the arteries become narrowed and hardened due to an excessive build up of plaque inside the artery wall (Nadeem *et al*., 2015). The formation of plaque disrupts the flow of blood around the body and leads to different cardiovascular diseases. According to the Global Burden of Disease Study, 17.3 million deaths across the world in 2013 were related to cardiovascular diseases (Sharma, 2015; Thompson, 2015). There was an increment of 41% since 1990. Undoubtedly, cardiovascular diseases are the leading cause of morbidity and mortality of our generation, more than 30% of all deaths in people aged 35 and above (Ponalagusamy, 2012). Atherosclerosis starts at a young age, with progressive plaque deposition in the major arteries of the body. When such a plaque becomes big enough in the coronary artery, myocardial ischemia or infarction can follow (Ajdari *et al*., 2017; Akbar, 2016). Since the pathogenesis of atherosclerosis starts at the cellular level, only an effective intervention at this level can thwart its progression. Atherosclerosis is a chronic inflammatory disease of the arterial wall that arises from an imbalanced lipid metabolism and a maladaptive inflammatory response (Akbar *et al*., 2015; Akbar *et al*., 2016). Despite intensive research on mechanisms underlying atherosclerotic lesion formation and progression during the past decade, translation of this knowledge into the clinic is scarce. Nanotechnology holds

tremendous potential to advance the current treatment of coronary artery disease (Akbar *et al*., 2016; Shit, 2015). Nanotechnology may assist medical therapies by providing a safe and efficacious delivery platform for a variety of drugs aimed at modulating lipid disorders, decreasing inflammation and angiogenesis within atherosclerotic plaques, and preventing plaque thrombosis. Nanotechnology may improve coronary stent applications by promoting endothelial recovery on a stent surface utilizing bio-mimetic nanofibrous scaffolds, and also by preventing in-stent restenosis using nanoparticle-based delivery of drugs that are decoupled from stents (Akbar *et al*., 2016; Akbar, 2016).

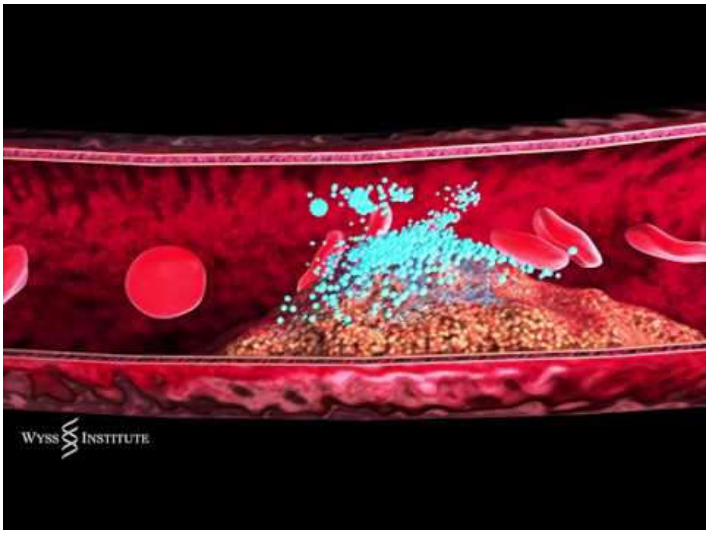

Fig. 1. Blood flow in stenosed artery

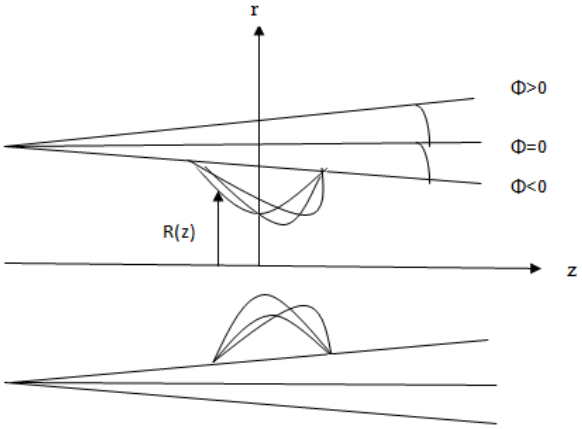

Fig. 2. Blood flow in a stenosed tapered artery

Additionally, nanotechnology may enhance tissue-engineered graft materials for application in coronary artery bypass grafting by facilitating cellular infiltration and remodeling of a graft matrix (Ellahi, 2014). Thus, it makes sense that nanomedicine can be an effective strategy for the treatment of Atherosclerosis. In this research work, we focus on the application of nanotechnology in the treatment of coronary artery disease (CAD) and highlight several areas of opportunity where nanotechnology may lead to novel classes of therapeutics or enhance existing therapies. Here we describe the features of currently available nanomedical formulations that have been optimized for atherosclerosis treatment, and we further describe how they can be instructed to target inflammatory processes in the arterial wall. Despite their limitations, nanomedical applications might hold promise for personalized medicine, and further efforts are needed to improve atherosclerosis-specific targeting.

PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

Let us consider one dimensional pulsatile, axially symmetric, laminar, incompressible, fully developed flow of blood is treated as Jeffrey fluid with nano-particles, having constant viscosity μ and density ρ, through a tube shaped artery of radius R_0 and length L (Thompson, 2015). The cylindrical coordinate system (r, θ, z) is chosen such that the velocity components in r and z directions is u and v respectively. It is assumed that the stenosis develops in the arterial wall in an axis non- symmetric but radially symmetric manner and depends upon the axially distance z and the height of its growth. Heat and mass transfer phenomenon is calculated by assigning temperature T_1 and concentration C_1 to the wall of the tube .The geometry of the arterial wall with overlapping stenosis is given as (Akbar *et al*., 2016):-

$$\frac{R(z)}{D(z)} = [1 - \psi(L_0^{n-1}(z - d_0) - (z - d_0)^n] \quad ; \qquad d_0 < z \leq d_0 + L_0$$

$$\frac{R(z)}{D(z)} = \begin{cases} 1 & \\ & \text{otherwise} \end{cases} \quad (1)$$

With

$$\psi = \frac{(\delta)n^{\frac{n}{n-1}}}{R_0 L_0^n (n-1)} \quad (2)$$

$$d(z) = R_0 + \xi z, \quad (3)$$

in which δ denotes the maximum height of the stenosis located at

$$z = d_0 + \frac{L_0}{n^{\frac{n}{n-1}}} \quad (4)$$

Here R_0 is the radius of the non-tapered artery in the non-stenotic region, $d(z)$ is the radius of the tapered arterial segment in the stenotic region, is the tapering parameter, L_0 is the length of the stenosis, $n(\geq 2)$ is a parameter determining the shape of the constriction profile, referred to as the stenosis shape parameter for which the symmetric stenosis is found for n=2 and d_0 indicates location of stenosis, as shown in Fig. (1,2).

The equations governing the flow are:

$$\frac{v}{r} + \frac{\partial(v)}{\partial r} + \frac{\partial u}{\partial z} = 0 \quad (5)$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) - \frac{1}{r}(S_{\theta\theta}), \quad (6)$$

$$\rho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) + \rho g\alpha_1(T - T_1) + \rho g\alpha_1(C - C_1) \quad (7)$$

$$\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = \alpha_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r} + \frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right) + \frac{D_T}{T_0}\left(\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right)\right] \quad (8)$$

$$\left(v\frac{\partial C}{\partial r} + u\frac{\partial C}{\partial z}\right) = D_B\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_0}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) \quad (9)$$

where p is pressure, g is the acceleration due to gravity ,T -is temperature, C- is concentration, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nanoparticle and heat capacity of the fluid. The ambient values of T and C as r tent to R are denoted by T_1 and C_1 , D_B is the Browning diffusion coefficient and D_T is the thermospheric diffusion coefficient.

$$S_{rr} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(v\frac{\partial}{\partial r} + u\frac{\partial}{\partial z}\right)\right)\frac{\partial v}{\partial r},$$

$$S_{rr} = \frac{\mu}{1+\lambda_1}\left(1 + \lambda_2\left(v\frac{\partial}{\partial r} + u\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right),$$

$$S_{zz} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(v\frac{\partial}{\partial r} + u\frac{\partial}{\partial z}\right)\right)\frac{\partial u}{\partial z},$$

where λ_1 is the ratio between relaxation to retardation times ,and λ_2 is the retardation time . Defining;

$$r' = \frac{r}{R_0}; \quad z' = \frac{z}{L_0}; \quad v' = \frac{L_0}{\delta U}; \quad u' = \frac{u}{U};$$

$$R' = \frac{R}{R_0}; \quad p' = \frac{R_0^2}{U\mu L_0}p; \quad \varphi = \frac{T - T_1}{T_0 - T_1}; \quad \sigma = \frac{C - C_1}{C_0 - C_1};$$

$$G_r = \frac{\rho g\alpha_1 R_0^2}{\mu}(T_0 - T_1); \quad B_r = \frac{\rho g\alpha_1 R_0^2}{\mu}(C_0 - C_1); \quad R_e = \frac{\rho U R_0}{\mu};$$

$$N_t = \frac{(\rho c)_p D_T T_0}{(\rho c)_f \alpha_1}; \quad N_b = \frac{(\rho c)_p D_B C_0}{(\rho c)_f \alpha_1}; \quad (10)$$

where R_e –is the Reynolds number , N_t –is the thermophoresis parameter , N_b –is the Brownian motion parameter , G_r –is the local temperature Grashof number , B_r –is the local Grashof number. Using the non-dimensional variables in equation (10) along with the additional boundary conditions.

$\frac{R_e \delta n^{\frac{1}{n-1}}}{L_0} \ll 1$,

$\delta^* = \frac{R_0 n^{\frac{1}{n-1}}}{L_0} \sim o(1)$,

and for mild stenosis $\left(\frac{\delta}{R_0} \ll 1\right)$ in equation (5) to (9), after dropping the dashes take the form

$$\delta^* \left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{\partial u}{\partial z} = 0 \quad , \tag{11}$$

$$\frac{\partial p}{\partial r} = 0 \quad , \tag{12}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\left(\frac{\partial u}{\partial r}\right)\right) + G_r \varphi + B_r \sigma \quad , \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) + N_b \frac{\partial \sigma}{\partial r}\frac{\partial \varphi}{\partial r} + N_t \left(\frac{\partial \varphi}{\partial r}\right)^2 = 0 \quad , \tag{14}$$

$$N_b \frac{\partial}{\partial r}\left(r\frac{\partial \sigma}{\partial r}\right) + N_t \frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) = 0 \tag{15}$$

After integration equation (15) gives the result

$$\sigma = -\varphi \frac{N_t}{N_b} \tag{16}$$

The boundary conditions are as follows:

$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial \varphi}{\partial r} = 0, \quad \frac{\partial \sigma}{\partial r} = 0$ at r=0

w=0, $\varphi = 0, \quad \sigma = 0$ at r=R(z),

where

$\frac{R(z)}{1+\xi_1 z} = [1 - \psi_1((z - d_0^*) - (z - d_0^*)^n)], \quad d_0^* < z \leq d_0^* + 1$,

$\frac{R(z)}{1+\xi_1 z} = 1$, otherwise, (17)

where

$d_0^* = \frac{d_0}{L_0}, \quad \xi_1 = \frac{\xi L_0}{R_0}, \quad \psi_1 = \delta^* \frac{n^{\frac{n}{n-1}}}{(n-1)}$.

Solution of the problem using numerical and analytical applied methods

Solution of the equation (14) are calculated by homotopy perturbation method as

$$H(k,\varphi) = (1-k)[L(\varphi) - L(\varphi)_{10}] + k\left[L(\varphi) + N_b \frac{\partial \sigma}{\partial r}\frac{\partial \varphi}{\partial r} + N_t \left(\frac{\partial \varphi}{\partial r}\right)^2\right], \tag{18}$$

where k is the embedding parameter, which has the range $0 \leq k \leq 1$, $L = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$, is a linear operator.
Taking the following initial guess

$$\varphi_{10}(r,z) = -\left(\frac{r^2-R^2}{4}\right) \tag{19}$$

Define

$$\varphi = \varphi_0 + k\varphi_1 + k^2\varphi_2 + o(k)^3 \tag{20}$$

Putting equations (20) in equation (14), and taking $k \to 1$, the following expression for temperature profile is obtained as follows:

$$\varphi(r,z) = (2N_t + N_b)\left(\frac{r^4-R^4}{64}\right) - \left(\frac{r^6-R^6}{1152}\right)(2N_t + N_b)(N_t + N_b) \tag{21}$$

Using the above result of temperature profile in equation (16), we get

$$\sigma(r,z) = \frac{N_t}{N_b}\left((2N_t + N_b)\left(\frac{r^4-R^4}{64}\right) - \left(\frac{r^6-R^6}{1152}\right)(2N_t + N_b)(N_t + N_b)\right) \tag{22}$$

By putting equation (21) and (22) in equation (13) we get the result for velocity profile as

$$u(r,z) = \frac{r^2}{2}(1+\lambda_1)\frac{dp}{dz} - \left(G_r - B_r\frac{N_t}{N_b}\right)(2N_t + N_b)(1+\lambda_1)\left(\frac{r^7-21r^3R^4}{8064} - \frac{r^9-12r^3R^6}{82944}\right) \tag{23}$$

RESULTS AND DISCUSSION

It is studied in the nature of blood in arteries as non-Newtonian fluid is investigated analytically. Homotopy perturbation method is applied to solve the temperature profile governing equation. The result of temperature profile is used to evaluate results for concentration and velocity profile. In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to evaluate the analytical results obtained for temperature, concentration and velocity profile. A significant influence of different prominent flow parameters on the blood flow with nanoparticles are taking into account by the graphs of temperature profile (φ), concentration profile (σ) and velocity profile u (r, z). The graphs of velocity profile u(r, z), concentration profile (σ) and temperature profile (φ) are discussed for different values of Grashof number (Gr), local Grashof number (Br), thermophoresis parameter (Nt) and Brownian motion parameter (Nb).

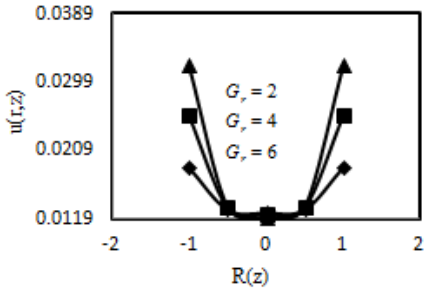

Fig.3 Variation of velocity profile for different values of Grashof number

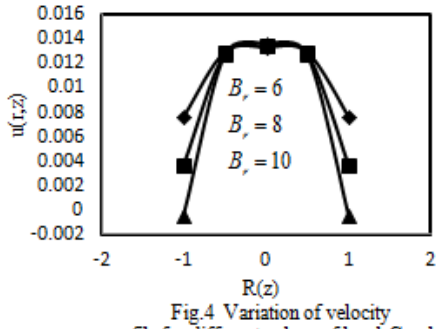

Fig.4 Variation of velocity profile for different values of local Grashof number

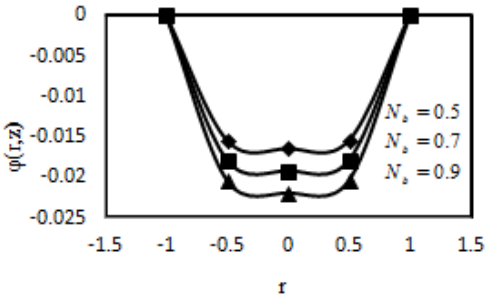

Fig.5 Variation of temperature profile for different values of Brownian motion parameter

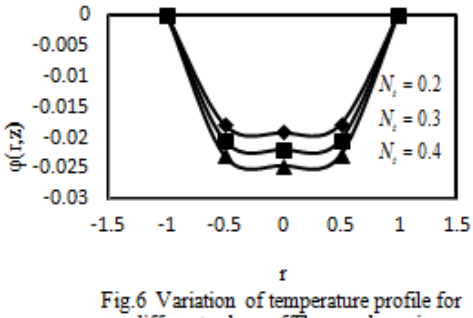

Fig.6 Variation of temperature profile for different values of Thermophoresis parameter

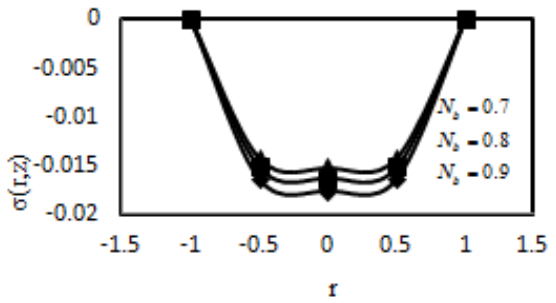

Fig.7 Variation of concentration profile for different values of Brownian motion parameter

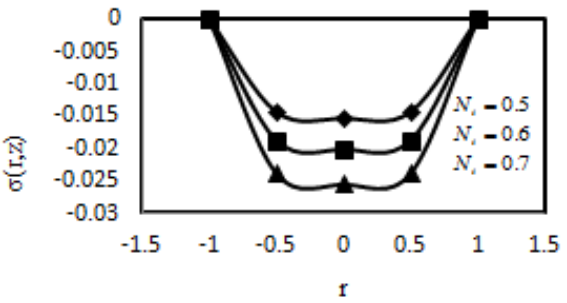

Fig.8 Variation of concentration profile for different values of thermophoresis parameter

Where fig. 3 depicts the variation of velocity profile u(r,z) with radius of artery with stenosis R(z) for different values of Grashof number Gr .It is observed in the figure that with increase in Grashof number Gr the velocity profile u(r,z) increases (Raza, 2016; Thompson, 2015). But interestingly it is found that at R(z) = 0 , the velocity profiles varies in different manner ,i.e. with increase in Grashof number the velocity profile decreases. Physical significance:- Grashof number expresses the ratio between the Buoyancy forces and viscous forces (Akbar, 2016). Hence Grashof number is inversely proportional to the viscous forces, which implies that with increases in Grashof number the viscous forces decreases. With decreases in viscous forces velocity profile will automatically increase. Where fig. 4 depicts the variation of velocity profile u(r,z) with radius of artery with stenosis R(z) for different values of local Grashof number (Br) . It is observed in the figure that with increase in local Grashof number Br the velocity profile u(r, z) decreases. But interestingly it is found that at R(z) = 0 , the velocity profiles varies in different manner ,i.e. with increase in local Grashof number Br the velocity profile increases (5). Physical significance:-Local Grashof number is directly proportional to the concentration. So with increases in local Grashof number result in increasing concentration of the fluid. With increases in concentration the viscosity start get increased and automatically it will result decrease in blood flow profile. Figure 5 shows the variation of temperature profile for different values of Brownian motion parameter. It is observed from the figure that with increases in Brownian motion parameter temperature profile decreases. Physical significance:- Brownian motion parameter concerns with the random motion of particles suspended in a fluid resulting from the collision with the fast moving red blood cells, white blood cells and other constituents in blood. With increases in Brownian motion parameter dimensionless temperature profile decreases, this means temperature of the fluid increases. Hence Brownian motion parameter directly corresponds to the temperature of the fluid. Fig. 6 shows the variation of temperature profile φ(r,z) for different values of thermophoresis parameter Nt . It is observed from the figure that with increase in thermophoresis parameter Nt temperature profile φ(r,z) decreases (Akbar, 2015; Akbar *et al*., 2016). Physical significance:- Thermophoresis is a phenomenon observed in mixtures of mobile particles where different particle types exhibit different responses to the force of temperature gradient. So with increase in thermophoresis parameter Nt dimensionless temperature profile decreases, which indicates that temperature of the fluid increases (Akbar, 2016; Akbar *et al*., 2016). Fig. 7 shows the variation of concentration profile σ(r,z) for different values of Brownian motion parameter Nb . It is observed in the figure that with increase in Brownian motion parameter Nb concentration profile increases. Physical significance:- Brownian motion parameter concerns with the random motion of particles suspended in a fluid resulting from the collision with the fast moving red blood cells , white blood cells and other constituents in blood. With increases in Brownian motion parameter dimensionless concentration profile decreases which shows that concentration of the fluid decreases. Fig. 8 the variation of concentration profile σ(r,z) for different values of thermophoresis parameter Nt. It is observed in the figure that with increase in thermophoresis parameter Nt concentration profile σ(r,z) decreases. Physical significance:- Thermophoresis is a phenomenon observed in mixtures of mobile particles where different particle types exhibit different responses to the force of temperature gradient. So with increase in thermophoresis parameter Nt dimensionless concentration profile decreases.

Conclusion

In this present model, metallic nanoparticles analysis trough an axisymmetric mild stenosis, with blood is considered as non-Newtonian fluid is investigated. The heat and mass transfer via nanoparticles are also taken into account. This process relies on understanding the detail of blood flow through arteries. The flow governing equations are solved using Homotopy Perturbation Method.

The main points of the performed analysis are as follows:-

- The velocity profile (u) decreases with an increase in the Grashof number (G_r) and local Grashof number (B_r).
- The temperature profile (φ) decreases with an increase in Brownian motion parameter (N_b) and thermophoresis parameter (N_t).
- The concentration profile (σ) decreases with an increase in the Brownian motion parameter (N_b) and thermophoresis parameter (N_t)

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