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EFFECT OF PARAMETERS ON SECURE OPTICAL COMMUNICATION BASED ON CHAOS SYNCHRONIZATION

*Jiuli Yin, Lishuang Bian and Lixin Tian

Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, China

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ABSTRACT

For further understanding optical secure communication, we study chaos synchronization modeled by the nonlinear Schrödinger equation with Kerr law nonlinearity. By Melnikov method, we get the chaos signal which generated from rupturing of homoclinic orbits. By using feedback control technology and Lyapunov stability theory, we obtain the sufficient criteria of of chaos synchronization. Finally, we analysis the effects of all parameters on synchronization. Result shows that faster wave speed, higher perturbation amplitude and larger nonlinear dispersion coefficient can delay the process of synchronization. Meanwhile, larger control coefficient can speed up the process of synchronization.

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INTRODUCTION

Optical communication is an important technology which has been applied to various aspects of society. Multimedia is applied by optical FFH-CDMA communication systems (Thiruchelvi *et al*., 2002). Ring launching scheme is analyzedby hollow optical fiber mode converter (Bunge *et al*., 2006). Data encryption of optical fibre communication is considered based on pseudo-random spatial light modulation (Kowalskiand Zyczkowski, 2016). Recently, large capacity and long distance are required in optical fiber communication. Optical soliton as the most ideal carriers of information just meets these demands. Optical soliton like-pulses is obtained in ring-cavity fiber lasers of carbon nanotubes (Younis and Rizvi, 2016), controllable behaviors of spatiotemporal breathers is considered in a generalized variable-coefficient nonlinear Schrödinger model from arterial mechanics and optical fibers (Chen and Zhu, 2015), transmission characteristics of dark solitons have been studied in homogeneous optical fibers modeled by nonlinear Schrödinger equation (Pan, *et al*., 2015). With the development of technology and military, traditional communication can not grantee the safety and effectiveness of the information transmission. So we want to study the optical secret communication. Chaos synchronization technology is an effective method to study secure communication. Long-distance multi-channel bidirectional chaos communication has been constructed by synchronized VCSELs subject to chaotic signal injection (Xie *et al*., 2016), Different types of synchronization in coupled network based chaotic circuits (Srinivasan, 2016), chaos synchronization is studied in two weakly coupled delay-line oscillators(Levy and Horowitz,2012), dynamics and synchronization are applied in coupled fractional-order nonlinear electromechanical systems (Ngueuteu *et al*.., 2012). The purpose of this paper is to devise optical secure communications based on the nonlinear Schrödinger equation by chaos synchronization. The inspiration comes from the fact as follows. In the process of the actual transmission, the signals are influenced by the external environmental perturbations, so we consider perturbed nonlinear Schrödinger equation with Kerr law nonlinearity

$$iu_t + u_{xx} + \alpha|u|^2 u + i[\gamma_1 u_{xxx} + \gamma_2 |u|^2 u_x + \gamma_3 (|u|^2)_x u] = \mu u \cos\omega(x - ct), \quad (1)$$

***Corresponding author:* Jiuli Yin,**
Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, China.

where $\gamma_i(i=1,2,3)$ are real parameters, is a positive constant, $\mu,\omega\geq 0$ denote the amplitude and the frequency of the parametric excitation, respectively. More details for the model can be seen in (Taghizadeh *et al*., 2013; Yin *et al*., 2014). This paper is organized as follows. In Section 2, we proved that the soliton can evolves into chaos under periodic perturbation proved by Melnikov method. In Section 3, we construct a master-salve synchronization scheme and find out the criteria for chaos synchronization. The numerical investigation is given in Section 4. Last is the conclusion.

2 Soliton evolves into chaos under perturbation

Supposed that (1) has traveling wave solutions in the form

$$\mu(x,t)=a(\xi)\exp(-i\Omega t),\quad \xi=x-ct, \tag{2}$$

here c representing the transmission speed of wave $(c>0)$.

By the way of (Taghizadeh *et al*., 2013),we get

$$\gamma_1 a'' - ca + \lambda a^3 = \mu a\cos(\omega\xi), \tag{3}$$

where $\lambda=\frac{1}{3}\gamma_2+\frac{2}{3}\gamma_3$ refer to the parametric of linear and nonlinear terms. Equation (3) is the fiber-optic signal transmission system in ideal environment.

By setting $\gamma_1=1$ and $b=a'$, the system (3) can be rewritten as the following form

$$\begin{cases} a'=b, \\ b'=(\mu\cos(\omega\xi)+c)a-\lambda a^3. \end{cases} \tag{4}$$

If $\mu=0$, the equation (4) is changed into (5)

$$\begin{cases} a'=b, \\ b'=ca-\lambda a^3. \end{cases} \tag{5}$$

Analysis of homoclinic orbits

We consider the homo clinic orbits of system (5), which has three equilibrium points $E_1(-\sqrt{\frac{c}{\lambda}},0)$, $E_2(\sqrt{\frac{c}{\lambda}},0)$ and $E_3(0,0)$. Let J_{E_i} be the Jacobian matrix for these equilibrium points and we have

$$J_{E_i}=\begin{bmatrix} 0 & 1 \\ c-3\lambda a^2 & 0 \end{bmatrix}.$$

It is easy to find that its eigenvalues are $\lambda_{1,2}(J_{E_i})=\pm\sqrt{-2c}$ and $\lambda_3(J_{E_3})=\pm\sqrt{c}$. So the E_1, E_2 are the center equilibrium and $E_3(0,0)$ is a saddle point for any $c>0$.

Furthermore, system (5) has the following Hamiltonian function:

$$H(a,b)=\frac{1}{2}a^2-\frac{1}{2}ca^2+\frac{1}{4}\lambda a^4=h, \tag{6}$$

where h is a constant. It is noted that the Hamiltonian function is composed of two homoclinic orbits at the point E_3. According to the bifurcation theory (13), system (5) has two optical solitons followed by two homoclinic orbits: Positive one achieve its crest at $\alpha=\sqrt{\frac{2c}{\lambda}}$, and negative one has valley at $\alpha=-\sqrt{\frac{2c}{\lambda}}$ (see Fig. 1 as $\lambda=0.96$ and $c=2.5$).

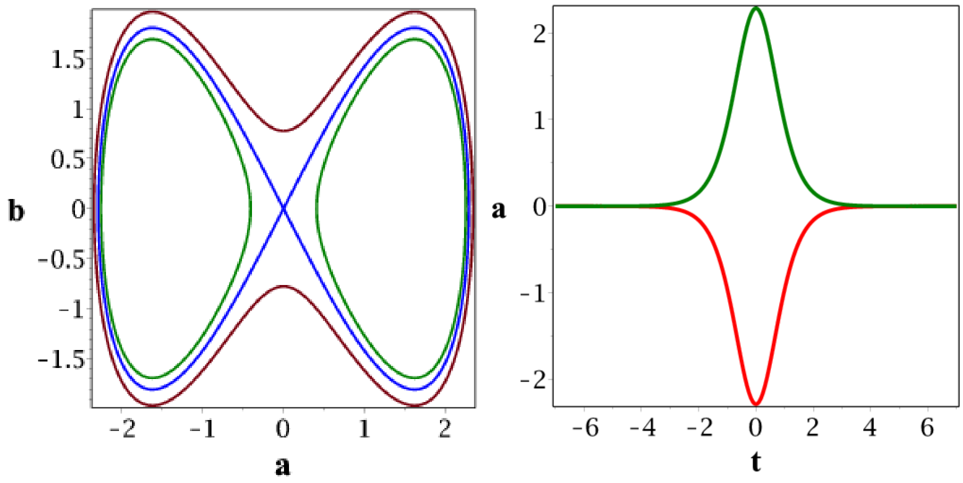


Figure 1. Phase portrait and the profile of optical soliton of unperturbed system

Melnikov analysis

For searching chaos signal, we will take advantage of the Melnikov's method. The corresponding Melnikov function for system (3) is given as

$$M(t_0)=\int_{-\infty}^{+\infty} b_0(t)\mu a_0(t)\cos(\omega(t+t_0))dt$$
$$=\frac{\mu}{2}\left[a_0^2(x)\cos(\omega(t+t_0))\Big|_{-\infty}^{+\infty}+\int_{-\infty}^{+\infty}\omega a_0^2(t)\sin(\omega(t+t_0))dt\right]$$
$$=\frac{\mu\omega}{2}\int_{-\infty}^{+\infty} a_0^2(t)\cos\omega t\sin\omega t_0 dt$$
$$=\frac{\mu\omega}{2}\sin\omega t_0 I, \tag{7}$$

where $I=\int_{-\infty}^{+\infty} a_0^2(t)\cos\omega t dt$ and $(a,b)=(a_0(t),b_0(t))$ is unperturbed homoclinic orbit. Using the above results and Melnikov's theorem, the chaos occurs if $M(t_0)=0$ and $M^{'}(t_0)\neq 0$ for some t_0. It is easy to find $t_0=0$ satisfying

$$\begin{cases}\sin(\omega t_0)=0,\\ \cos(\omega t_0)\neq 0.\end{cases} \tag{8}$$

So the optical soliton always turns into chaos under the perturbation formed as (1). To verify the above fact, we will investigate the Lyapunov exponents and phase portraits of system (4). Parameters of system (4) used for simulations are listed as follows: $c=2.5, \lambda=0.98, \mu=-0.5$ and $\omega=0.17$. Then the Lyapunov exponents are shown in Fig. 2(a) and the phase portraits are shown in Fig. 2(b), It is clearly that chaos always appears for all given cases.

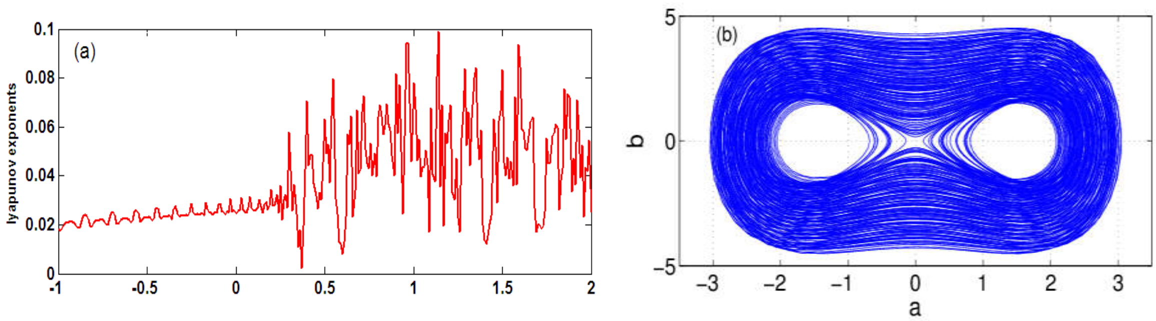


Figure 2. (a) **Lyapunov exponents versus** μ **of perturbed system,** (b) **The corresponding portraits of** $a-b$.

Global chaos synchronization method

Construction of error system

The equivalent vector form of chaotic oscillator (3)as flows

$$X' = A(t)x + f(x), \tag{9}$$

with $X = (x_1, x_2)^T \in R^2$,

$$A(t) = \begin{pmatrix} 0 & 1 \\ \delta(t) & 0 \end{pmatrix}, \qquad f(x) = \begin{pmatrix} 0 \\ -\lambda x^3 \end{pmatrix}, \tag{10}$$

where $\delta(t) = \mu\cos(\omega t) + c$. Next we construct a master-salve synchronization scheme (Wu, 2008) for two chaos oscillators coupled by a linear state error feedback controller $u(t) = K(x - z)$:

$$\begin{cases} M : X' = A(t)x + f(x), \\ S : Z' = A(t)z + f(z) + \mu(t), \end{cases} \tag{11}$$

where $Z = (z_1, z_2)^T \in R^2$,and $K \in R^{2\times 2}$ denotes a constant control matrix.
Define an error variable $e = x - z$. since

$$f(x) - f(z) = \begin{pmatrix} 0 & 0 \\ -\lambda F(t) & 0 \end{pmatrix} e = M(t)e, \tag{12}$$

With $F(t) = x_1^2 + x_1 z_1 + z_1^2$, we can obtain a time -varied error system

$$\dot{e} = \dot{x} - \dot{z} = [A(t) - K]e + f(x) - f(z) = [A(t) + M(t) - K]e. \tag{13}$$

Next, we are seeking to find such a control matrix K on any initial conditions of $x(0)$ and $z(0)$, the trajectories $x(t)$ and $z(t)$ satisfy

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|x(t) - z(t)\| = 0, \tag{14}$$

Where $\|\cdot\|$ denotes Euclidean norm of a vector .

Criteria for chaos synchronization

According to Lyapunov's direct method we get the criteria for chaos synchronization as follows.

Lemma. If there exists a symmetric $0 < P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{12} \end{pmatrix} \in R^{2\times 2}$ and a control matrix $K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{12} \end{pmatrix} \in R^{2\times 2}$ where

$$H_1 = -p_{11}k_{11} - p_{12}k_{21} + |p_{12}|(|u| + |c| + 3|\lambda| m^2) < 0. \tag{15}$$

$$H_2 = p_{12}(1 - k_{12}) - p_{22}k_{22} < 0. \tag{16}$$

$$4\mathrm{H}_1\mathrm{H}_2 > [p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22}) - p_{22}k_{21} + p_{22}(|u| + |c| + 3|\lambda| m^2)]^2. \tag{17}$$

Then the master-salve scheme (11) achieves chaos synchronization .

Proof . Take a quadratic Lyapunov function

$V(e) = e^T Pe$, with $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} > 0$.

The derivative of $V(e)$ with respect to time along the trajectory of error system (13) equals

$$\dot{V}(e) = (\dot{e})^T \ Pe + e^T P\dot{e} = [(A(t) + M(t) - K)e]^T Pe + e^T P[(A(t) + M(t) - K)e]$$
$$= e^T [(A(t) + M(t) - K)^T P + P(A(t) + M(t) - K)]e.$$

$$\dot{V}(e) < 0 \quad if$$
$$Y = (A(t) + M(t) - K)^T P + P(A(t) + M(t) - K) < 0, \quad \forall t \geq 0 \tag{18}$$

According to Lyapunov stability theory, the inequality (16) represents a sufficient condition for global asymptotic stability of error system (13) at the origin. According to (10) and (12), we can obtain

$$Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix},$$

where

$$Y_{11} = -2p_{11}k_{11} + 2p_{12}(\delta - \lambda F - k_{21}),$$
$$Y_{12} = p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22}) + p_{22}(\delta - \lambda F - k_{21}),$$
$$Y_{21} = p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22}) + p_{22}(\delta - \lambda F - k_{21}),$$
$$Y_{22} = -2p_{22}k_{22} + 2p_{12}(1 - k_{12}).$$

Since Y is symmetric. $Y < 0$ if and only if $Y_{11} < 0, Y_{22} < 0, |Y| > 0.$

From Fig.2 (b),we know that the trajectory of chaotic system (3) is bounded, so there exists a constant $m > 0$ satisfy $|x_1(t)| < m$ for any $t \geq 0$. Hence , we have

$$|\delta(t)| = |\mu \cos(\omega t) + c| \leq |\mu| + |c|, \quad |F(t)| = |x_1^2 + x_1 z_1 + z_1^2| \leq 3m^2.$$

Again $p_{ii} > 0 (i = 1,2)$ on condition that $P > 0,$ we have

$$-2p_{11}k_{11} + 2p_{12}(\delta - \lambda F - k_{21}) \leq -2p_{11}k_{11} - 2p_{12}k_{21} + |2p_{12}(\delta - \lambda F)| \leq 2H_1,$$
$$|p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22}) + p_{22}(\delta - \lambda F - k_{21})| \leq |p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22}) - p_{22}k_{21}|$$
$$+ p_{22}(|\mu| + |c| + 3|\lambda| m^2).$$

So the inequalities (15)-(17) hold . The proof is finished.#

Numerical simulations

In applications, it is desired that the structure of synchronization controller is as simple as possible. So we find out the two special cases for the Lemma.

Case 1 For the inequalities (15)-(17) , setting $k_{11} = k_1, k_{22} = k_2$ and $k_{12} = k_{21} = 0,$ then we have the inequalities (19)-(21).

$$k_1 > \frac{|p_{12}|(|\mu| + |c| + 3|\lambda| m^2)}{p_{11}}, \tag{19}$$

$$k_2 > \frac{p_{12}}{p_{22}}, \tag{20}$$

$$4[|p_{12}|(|\mu| + |c| + 3|\lambda| m^2) - k_1 p_{11}][p_{12} - k_2 p_{22}] - [p_{11} - p_{12}(k_1 + k_2) + p_{22}(|\mu| + |c| + 3|\lambda| m^2)]^2 > 0, \tag{21}$$

Here we choose an control matrix $K = diag\{k_1, k_2\}$ and a symmetric positive define matrix P selected like the Lemma. Then the system (11) can also achieve global chaos synchronization.

Case2 For the inequalities (19)-(21),setting $k_1 = k_2 = k$, then we have the inequalities (22)-(23).

$$k > \max\left\{\frac{|p_{12}|(|\mu|+|c|+3|\lambda|m^2)}{p_{11}}, \frac{p_{12}}{p_{22}}\right\} \geq 0 \tag{22}$$

$$4k^2(p_{11}p_{22} - p_{12}^2) + 4p_{12}^2((|\mu|+|c|+3|\lambda|m^2)) - [p_{11} + p_{22}(|\mu|+|c|+3|\lambda|m^2)]^2 > 0 \tag{23}$$

Herewe choose an control matrix $K = diag\{k, k\}$ and a symmetric positive define matrix P selected like the Lemma. Then the system (11) can also achieve global chaos synchronization. Since $p_{11}p_{22} - p_{12}^2 > 0$, the solution k to inequality (23) exists. We select $P_{12} = 0$ and $P_{11} = P_{22}(|\mu|+|c|+3|\lambda|m^2) > 0$ to construct a symmetric positive define $P = p_{22}\begin{pmatrix} |\mu|+|c|+3|\lambda|m^2 & 0 \\ 0 & 1 \end{pmatrix} > 0$. Using this matrix we can obtain the flowing algebraic synchronization criterion by (22) and (23).

$$K = diag\{k, k\}, \text{and } k > \sqrt{|\mu|+|c|+3|\lambda|m^2} > 0. \tag{24}$$

According calculate ,we obtain the synchronization condition :

$$K = diag\{k, k\}, k > 8.746.$$

Now we analyze the performance of chaotic secure communication system, the system (13) was simulated in Matlab-Simulink and the result are shown in Fig3-fig7.

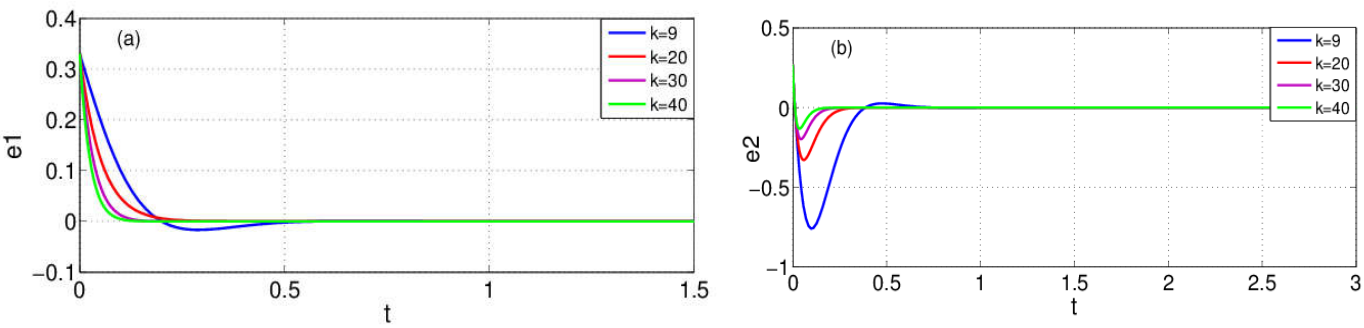


Figure 3. Effect of varying k. (a) error $e1$; (b) error $e2$.

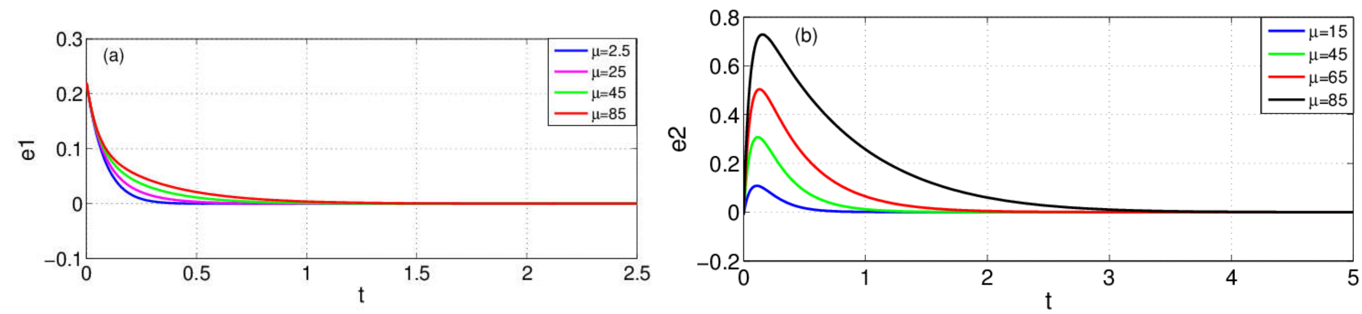
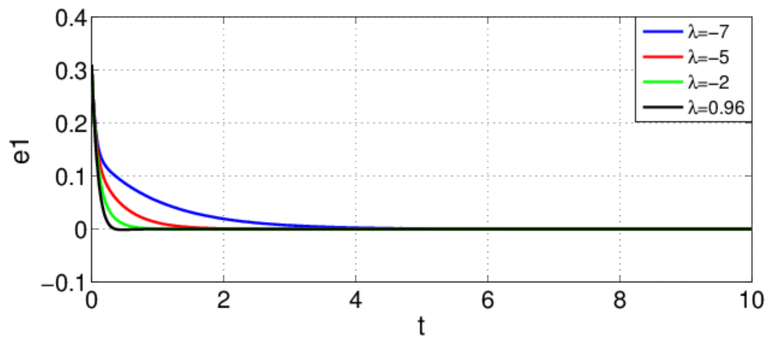


Figure. 4 Effect of varying μ. (a) error $e1$; (b) error $e2$.



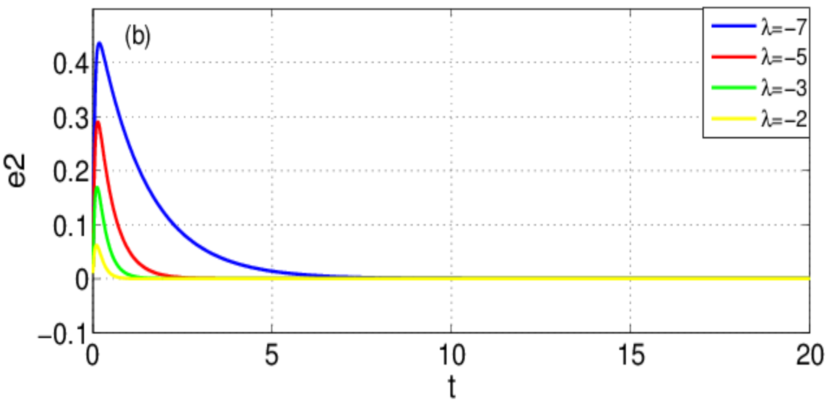


Figure 5. Effect of varying λ. (a) **error** $e1$; (b) **error** $e2$.

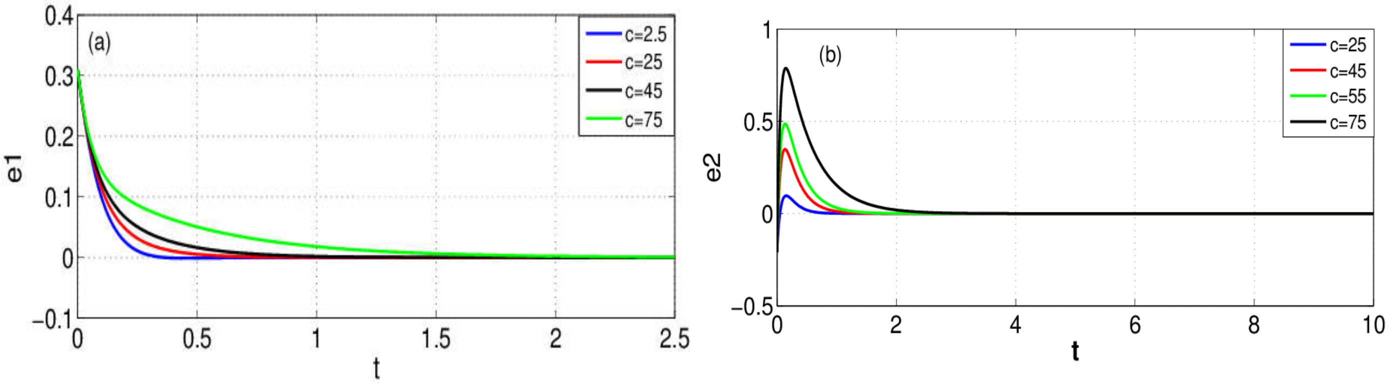


Figure 6. Effect of varying c. **(*a*) error** $e1$; **(*b*) error** $e2$.

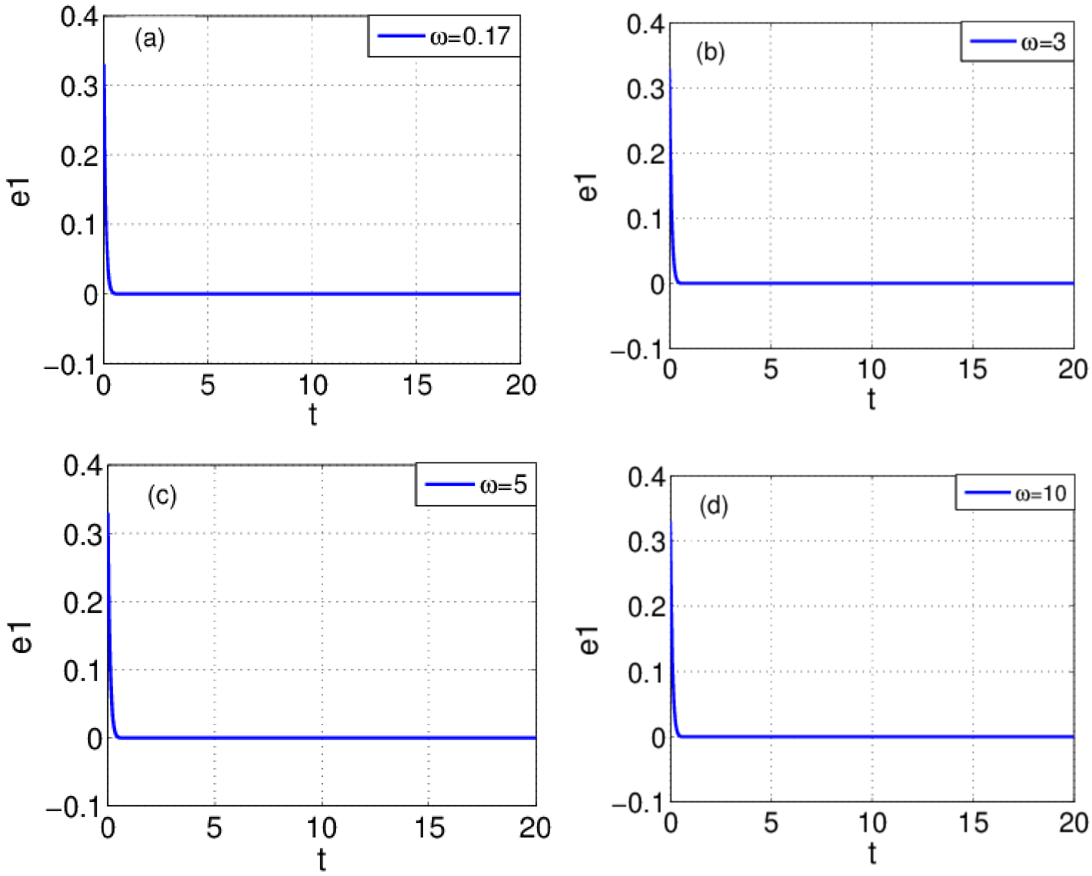


Figure 7. Effect of varying ω. (a) **error** $e1$; (b) **error** $e1$; (c) **error** $e1$; (d) **error** $e1$.

Conclusion

In this paper we have proposed a new chaotic secure communication scheme. The sufficient criteria for chaos synchronization of the master-salve system are obtained. Numerical simulations have demonstrated the accuracy of those sufficient criteria.

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