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RANDOM ITERATION METHODS FOR STRICTLY PSEUDO-CONTRACTIVE OPERATORS

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ABSTRACT

The aim of this chapter is to establish the convergence and almost stability results of random xed point iterative process using two strictly pseudo contractive random operators in a real separable Hilbert space. The results presented in this paper generalize several well known results in Hilberts spaces. Mathematics Subject Classification (2010) : 47H05, 47H09,47H10, 47J05.

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INTRODUCTION

We recall that Probabilistic functional analysis has come out as one of the momentous mathematical disciplines in view of its requirements in dealing with probabilistic models in applied problems. The study of random xed points forms a central topic in this area. Random xed point theory has received much attention since the publication of the survey article by Bharucha-Reid (?) in 1976, in which the stochastic version of some well-known xed point theorems were proved. Since then there has been a lot of activity in this area. For example, Li (20, ?, 18) has introduced the random xed point index theory and obtained some excellent random xed point theorems which are applied to investigate the existence of random solutions for random Hammerstein equation (see (21, 22) for details). Moreover, in 1979 Itoh (23), 1998 ORegan (24) and Shahzad (25, 26) studied many random xed points of contractive random maps satisfying di erent conditions. Choudhury (27) constructed a random Mann iteration scheme in a separable Hilbert space and proved a random xed point theorem satisfying a certain contractive inequality. In conclusion, random xed point theorems in connection with random approximations are studied extensively. In this paper, we rst introduce some new concepts such as random monotone operators, random Mann iteration and so on, which are a little di erent from those in (27). Moreover, we consider random operators satisfying Condition(H) and prove some new random xed theorems on these operators by virtue of monotone iterative methods and partial ordered theory. In fact, we also obtain the uniqueness of random xed points, which almost can not be obtained by the methods in previous literatures. In linear spaces there are two general iterations which have been successfully ap-plied to xed point problems of operators and also for obtaining solutions of operator equations.

These are Ishikawa iteration scheme and Mann iteration scheme (7). On the other hand, random xed point theory has attracted much attention in recent times es-pecially after the publication of the review article by Bharucha-Reid (?). We note some important recent works on random xed points in ((11, 12, 13, 14). In an attempt to construct iterations for nding xed points of random operators de ned on linear spaces, random Ishikawa iteration scheme was introduced in (15). This iteration and also some other random iterations based on the same ideas have been applied for nding solutions of random operator equations and xed points of random operators (see (15, 16, 17). The purpose of the paper is to introduce another random xed-point iteration that is a random Mann iteration scheme and to show that the random iteration if convergent, will under certain conditions converge to a random xed point of a random operator de ned in the context of a separable Hilbert space.

Preliminaries

We first review the following concepts which are essentials for our study in this chapter. Throughout this paper, $(\omega;\Sigma)$ denotes a measurable space and H stands for a separable Hilbert space. K is a nonempty subset of H. A function T: $\rightarrow K$ is said to be measurable if T^{-1} $(B \cap K)$ Σ for every Borel subset B of H.

A function T: $\Omega \times K \rightarrow K$ is said to be a random operator, if F (., $\xi(\omega)$) : $\Omega \rightarrow K$ is measurable for every $\xi(\omega)$ K.

A measurable function $\xi(\omega)$: $\Omega \rightarrow K$ is said to be a random fixed point of the random operator $T : \Omega \times X \rightarrow X$, if $T(\omega; \xi(\omega)) = \xi(\omega)$ for all $\omega \in \Omega$.

A random operator T: $\Omega \times K \rightarrow K$ is said to be continuous if, for fixed $\omega \in \Omega$, $T(\omega.,) : K \rightarrow K$ is continuous.

Recall that the normal Manns iterative process was introduced by Mann (7) in 1953. Since then, the construction of fixed points for non-expansive mappings and strict pseudo-contractions via the normal Manns iterative scheme has been extensively in vestigated by many authors. The normal Manns random iterative scheme generates a sequence in the sense of a random sequences is as follows:

$$\xi_{n+1}(\omega) = (1-\alpha_n)x_n(\omega) + \alpha_n T(\omega, x_n(\omega)), n \in \mathbb{N} \tag{2.1}$$

for each $\omega \in \Omega$ Ω, where $\{\alpha_n\}$ is in $(0;1)$. The Mann iteration method (2.1) for non expansive mappings and strict pseudo-contractions so that strong convergence is guar anteed have recently been made; see, (2, 3, 4, 5, 8, 9) and the references therein.

Kim and Xu (3) introduced the following iteration process

$$\begin{cases} x_1 = x_0 \in K, \\ x_{n+1} = (1-\alpha_n)y_n + \alpha_n u, \\ y_n = \beta_n x_n + (1-\beta_n)Tx_n, n = 0, 1, 2, \ldots \end{cases} \tag{2.2}$$

where T is a non-expansive mapping of K into itself, $u \in K$ is a given point. They proved the sequence $\{x_n\}$ defined by (2.2) converges strongly to a fixed point of T provided the control sequences $\{\alpha_n\}$ $\overset{\infty}{n}= 0$ and $\{\beta n\}$ $\overset{\infty}{n}= 0$ satisfy appropriate conditions Recently, Yao et al. (9) improved the results of Kim and Xu (3) by using the so-called viscosity approximation methods. To be more precisely, they introduced the following iterative scheme

$$\begin{cases} x_1 = x_0 \in K, \\ x_{n+1} = \alpha_n f(x_n) + (1-\alpha_n)y_n, \\ y_n = \beta_n x_n + (1-\beta_n)Tx_n, n = 0, 1, 2, \ldots \end{cases} \tag{2.3}$$

where f is a contraction on f. They obtained a strong convergence theorem for a non expansive mapping in a Banach space

Lemma 2.1 *(Zhou (6)). Let K be a nonempty subset of a real 2-uniformly smooth Banach spaces and $T : K \rightarrow K$ be a λ-strict pseudo-contraction. For $\alpha_n \in (0, 1)$, we define $T\alpha_x = (1-\alpha)x + \alpha T x$. Then as $\alpha \in (0, \frac{\lambda}{M2})$, T_α: $K \rightarrow K$ is non-expansive such that $F(T_\alpha) = F(T)$.*

Lemma 2.2 In a Banach space E, there holds the inequality

$$|x+y\|^2 \leq \|x\|^2 + 2\langle y, j(x+y)\rangle, x, y \in X,$$

where $j(x + y) \in J(x + y)$.

Lemma 2.3 *Assume that* $\{\alpha_n\}$ *is a sequence of nonnegative real numbers such that*

$\alpha_{n+1} \leq (1 - \gamma_n)\alpha_n + \delta_n,$

where $\{\gamma_n\}$ is a sequence in (0, 1) and $\{\delta_n\}$ is a sequence such that

(i) $\sum_{n=1}^{\infty} \gamma_n = \infty,$

(ii) $\limsup_{n\to\infty} \frac{\delta_n}{\gamma_n}$ *or* $\sum_{n=1}^{\infty} |\delta_n| < \infty.$

Then $\lim_{n\to\infty} \alpha n = 0.$

Lemma 2.4 (Zhou (6)). Let X be a smooth Banach space and K be a nonempty convex subset of X . Given an integer $r \geq 1$, assume that for each T: K → K is a λ_i-strict pseudo-contraction for some $0 \leq \lambda < 1$. As- sume that $\mu_i^r = 1$ is a positive sequence such that $\sum_{i=1}^{r} \mu_i = 1$. *Then* $\sum_{i=1}^{r} \mu_i T_i : K \to K$ *is a* λ_i - strict pseudo-contraction with $\lambda = \min\{\lambda_i : 1 \leq i \leq r\}.$

Lemma 2.5 *(Zhou (6)). Let X be a smooth Banach space and K be a nonempty convex subset of X. Given an integer* $r \geq$ *assume that* $\{T_i\}_i^r = 1$: $K \to K$ *is a finite family of* λi*-strict pseudo-contractions for some* $0 \leq \lambda < 1$ *such that* $F = \cap_i^r = 1$ $F(T_i) \neq \emptyset$. *Assume that* $\mu_i^r = 1$ *is a positive sequence such that* $\sum_{i=1}^{r} \mu_i = 1$. *Then* $F(\sum_{i=1}^{r} \mu_i T_i) = F.$

Convergence Results of a Random iteration Process

In this section, we investigate the convergence of two-step random iterative process for two nonexpansive random operators to obtain the random solution of the common random fixed point. This iterative process includes two-step random iterative process for a random operator *T* as special case.

Theorem 3.1 : *Let T be a random operator defined on a nonempty, closed, convex, subset K of a separable Hilbert space H. Let* $T : \Omega \times X \to X$ *be a* λ*−strict random pseudo-contraction such that* $F(T) \,\mathfrak{f}= \varphi$. *Given* $f \in {}_K$ *and for each* $\omega \in \Omega$ *and sequences* $\alpha_n(\omega)$ *and* $\beta_n(\omega)$, *the following control conditions are satisfied:*

(i) $\lim_{n\to\infty} \alpha_n(\omega) = 0,$

(ii) $\sum_{n=1}^{\infty} \alpha_n(\omega) = \infty,$

(iii) $0 < \beta_n(\omega) < \frac{\lambda}{K^2}$, *for all* $n \geq 0,$

(iv) $\sum_{n=1}^{\infty} |\alpha_{n+1}(\omega) - \alpha_n(\omega)| < \infty, \sum_{n=1}^{\infty} |\beta_{n+1}(\omega) - \beta_n(\omega)| < \infty.$

Then $\{\xi_n(\omega)\}$ *generated by*

$$\begin{aligned} \xi_{n+1}(\omega) &= \alpha_n(\omega) f(\omega, \xi_n(\omega)) + (1 - \alpha_n(\omega)\eta_n(\omega), \\ \eta_n(\omega) &= \beta_n(\omega) T(\omega, \xi(\omega)) + (1 - \beta_n)(\omega)\xi(\omega) \end{aligned} \tag{3.1}$$

Then $\{\xi_n(\omega)\}$ *converges strongly to a random fixed point of T .*

Proof : By Lemma 2.1, we have $\eta_n = T_{\beta n(\omega)}\xi_n$, $= F(T)$ and is $T_{\beta n(\omega)}$ non-expansive for every $\omega \in \Omega$. First, we show $\{\xi_n(\omega)\}$ is bounded. Indeed, taking a point $\xi(\omega) \in F(T)$, we have

$\|\eta_n(\omega) - \xi(\omega)\| \leq \|\xi_n(\omega) - \xi(\omega)\|.$

It implies that

$$\begin{aligned} \|\xi_{n+1}(\omega) - \xi(\omega)\| &= \|\alpha_n(\omega) f(\omega, \xi_n(\omega)) + (1 - \alpha_n(\omega)\eta_n(\omega) - \xi(\omega)\| \\ &= \|\alpha_n(\omega)(f(\omega, \xi_n(\omega)) - \xi(\omega)) + (1 - \alpha_n(\omega)(\eta_n(\omega) - \xi(\omega))\| \\ &\leq \alpha_n(\omega)[\|f(\omega, \xi_n(\omega)) - f(\omega, \xi(\omega)) \\ &\quad + f(\omega, \xi(\omega)) - \xi(\omega))\|] + (1 - \alpha_n(\omega))\|\eta_n(\omega) - \xi(\omega))\| \\ &\leq [1 - \alpha_n(\omega)(1 - \alpha_n(\omega))]\|\xi_n(\omega)) - \xi(\omega)\| + \alpha_n(\omega)\|f(\omega, \xi(\omega)) - \xi(\omega))\|. \end{aligned}$$

By induction method, we obtain

$$\|\xi_n(\omega)) - \xi(\omega)\| \le \max\{\|\xi_0(\omega)) - \xi(\omega)\|, \frac{\|f(\omega, \xi(\omega)) - \xi(\omega))\|}{1 - \alpha_n(\omega))}\},$$

for each $\omega \in \Omega$ and $n \ge 1$. Hence above condition gives that the sequence $\{\xi_n(\omega)\}$ is bounded. Similarly, we conclude that

$$\begin{aligned} \|\eta_{n+1}(\omega) - \eta_n(\omega)\| &= \|T_{\beta_{n+1}(\omega)}\xi_{n+1}(\omega) - T_{\beta_n}\xi_n(\omega)\| \\ &= \|T_{\beta_{n+1}(\omega)}\xi_{n+1}(\omega) - T_{\beta_{n+1}(\omega)}\xi_n(\omega)\| + \|T_{\beta_{n+1}(\omega)}\xi_n(\omega) - T_{\beta_n}\xi_n(\omega)\| \\ &\le \|\xi_{n+1}(\omega) - \xi_n(\omega)\| + \|T_{\beta_{n+1}(\omega)}\xi_n(\omega) - T_{\beta_n}\xi_n(\omega)\| \\ &\le \|\xi_{n+1}(\omega) - \xi_n(\omega)\| + M_1|\beta_{n+1}(\omega) - \beta_n(\omega)|, \end{aligned} \tag{3.2}$$

where M_1 is an appropriate constant such that $M_1 \ge \sup\{"T(\omega, \xi_n(\omega)) - \xi_n(\omega)"\}$. Ob- serving that

$$\begin{aligned} \xi_{n+2}(\omega) - \xi_{n+1}(\omega) = (1 - \alpha_{n+1}(\omega))(\eta_{n+1}(\omega) - \eta_n(\omega)) - (\alpha_{n+1}(\omega) - \alpha_n(\omega))\eta_n(\omega) \\ + \alpha_{n+1}(\omega)(f(\omega, xi_{n+1}(\omega)) - f(\xi_n(\omega))) \\ + f(\omega, \xi_n(\omega))(\alpha_{n+1}(\omega) - \alpha_n(\omega)), \end{aligned}$$

we get,

$$\begin{aligned} \xi_{n+2}(\omega) - \xi_{n+1}(\omega) = (1 - \alpha_{n+1}(\omega))(\eta_{n+1}(\omega) - \eta_n(\omega)) - |\alpha_{n+1}(\omega) - \alpha_n(\omega)|M_2 \\ + \alpha_{n+1}(\omega)\alpha_n(\omega)"\xi_{n+1}(\omega)) - \xi_n(\omega)" \end{aligned} \tag{3.3}$$

M_3 is a appropriate constant such that $M_3 \ge \max\{M_1, M_2\}$. Noticing condi- tions (i), (ii) and (iv) and applying Lemma 2.2 to (3.3), we obtain

$$\lim_{n\to\infty} \|\xi_n(\omega) - \xi_{n+1}(\omega)\| = 0. \tag{3.4}$$

Notice that

$$\begin{aligned} \|\xi_n(\omega) - \eta_n(\omega)\| &\le \|\xi_n(\omega) - \xi_{n+1}(\omega)\| + \|\xi_{n+1}(\omega) - \eta_n(\omega)\| \\ &\le \|\xi_n(\omega) - \xi_{n+1}(\omega)\| + \alpha_n(\omega)\|f(\xi_n(\omega)) - \eta_n(\omega)\|. \end{aligned}$$

It implies that, from condition (i) and (3.4), that

$$\lim_{n\to\infty} \|\xi_n(\omega) - T_{\beta_n(\omega)}(\omega, \xi_n(\omega)\| = 0. \tag{3.5}$$

Next, we claim that

$$\limsup_{n\to\infty} \langle f(\omega, \xi(\omega)) - \xi(\omega), J(\xi_n(\omega) - \xi(\omega))\rangle \le 0, \tag{3.6}$$

for each $\omega \in \Omega$, where $\xi(\omega) = \lim_{t\to 0} \xi_t$ being the fixed point of the contraction

$$\xi \mapsto tf(\omega, \xi(\omega)) + (1 - t)T_{\beta n(\omega)}(\omega, \xi(\omega)).$$

Then ξ_t solves the fixed point equation $\xi_t = t\gamma(\omega)f(\omega, \xi_t(\omega)) + (1 - t)T_{\beta n(\omega)}(\omega, \xi_t(\omega))$. Thus we get

$$\|\xi_t - \xi_n\| = \|(1 - t)(T_{\beta_n(\omega)}\xi_t - \xi_n) + t(f(\omega, \xi_t(\omega)) - \xi_n(\omega)\|.$$

It follows from Lemma 2.2 that

$$\begin{aligned} \|\xi_t(\omega) - \xi_n(\omega)\|^2 &= \|(1 - t)(T_{\beta_n(\omega)}(\omega, \xi_t(\omega)) - \xi_n(\omega)) + t(f(\xi_t(\omega)) - \xi_n(\omega))\|^2 \\ &\le (1 - t)^2\|T_{\beta_n(\omega)}(\omega, \xi_t(\omega)) - \xi_n(\omega)\|^2 \\ &\quad + 2t\langle f(\xi_t(\omega)) - \xi_n(\omega), J(\xi_t(\omega) - \xi_n(\omega))\rangle \\ &\le (1 - 2t + t)^2)\|\xi_t(\omega) - \xi_n(\omega)\|^2 + f_n(\omega)(t) \\ &\quad + 2t\langle f(\xi_t(\omega)) - \xi_t(\omega), J(\xi_t(\omega) - \xi_n(\omega))\rangle \\ &\quad + 2t\|\xi_t(\omega) - \xi_n(\omega), J(\xi_t(\omega) - \xi_n(\omega))\rangle, \end{aligned} \tag{3.7}$$

where

$$fn(\omega) = (2\|\xi_t(\omega) - \xi_n(\omega)\| + \|\xi_n(\omega) - T_{\beta_n(\omega)}(\omega, \xi_n(\omega))\|)\|\xi_n(\omega) - T_{\beta_n(\omega)}(\omega, \xi_n(\omega))\| \to 0, \tag{3.8}$$

as n → ∞ for each ω ∈ Ω. It gives from (3.7) that

$$2t\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), \xi_t(\omega) - \xi_n(\omega)\rangle \le t^2\|\xi_t(\omega) - \xi_n(\omega)\|^2 + f_n(t).$$

That is,

$$\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), J(\xi_t(\omega) - \xi_n(\omega))\rangle \le \frac{t}{2}\|\xi_t(\omega) - \xi_n(\omega)\|^2 + \frac{1}{2t}f_n(t). \tag{3.9}$$

Let $n \to \infty$ in (3.9) and note that (3.8) implies that

$$\limsup_{n\to\infty}\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), J(\xi_t(\omega) - \xi_n(\omega))\rangle \le \frac{t}{2}M_4 \tag{3.10}$$

where $M_4 > 0$ is a constant such that $M_4 \le \|xt.xn\|^2$ for all $t \in (0, 1)$ and $n \ge 1$. Taking $t \to 0$ from (refmainpseq10), we have

$$\limsup_{n\to\infty}\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), J(\xi_t(\omega) - \xi_n(\omega))\rangle \le 0.$$

Since X is uniformly smooth, J: $X \to X$ is uniformly continuous on any bounded sets of X, which ensures that the order of $\limsup_{t\to 0}$ and $\limsup_{t\to 0}$ is exchangeable, and hence

(3.6) holds. Now from Lemma 2.1, we have

$$\begin{aligned}
\|\xi_{n+1}(\omega) - \xi(\omega)\|^2 &= k(1 - \alpha_n(\omega))(\eta_n(\omega) - \xi(\omega)) \\
&\quad + \alpha_n(\omega)(f(\omega, \xi_n(\omega)) - \xi(\omega))\|^2 \\
&\le \|(1 - \alpha_n(\omega))(yn - \xi(\omega))\|^2 + 2\alpha_n(\omega)\langle f(\omega, \xi_n(\omega)) \\
&\quad - \xi(\omega), J(\xi_{n+1}(\omega) - \xi(\omega))\rangle \\
&\le (1 - \alpha_n(\omega))^2\|\xi_n(\omega) - \xi(\omega)\|^2 \\
&\quad + \alpha_n(\omega)\alpha(\omega)(\|\xi_n(\omega) - \xi(\omega)\|^2 + \|\xi_{n+1}(\omega) - \xi(\omega)\|^2) \\
&\quad + 2\alpha_n(\omega)\langle f(\omega, \xi(\omega)) - \xi(\omega), J(\xi_{n+1}(\omega) - \xi(\omega))\rangle,
\end{aligned}$$

which follows that

$$\begin{aligned}
\|x_{n+1} - \xi(\omega)\|^2 &\le \frac{(1 - \alpha_n(\omega))^2 + \alpha_n(\omega)\alpha(\omega)}{1 - \alpha_n(\omega)\alpha}\|\xi_n(\omega) - \xi(\omega)\|^2 \\
&\quad \frac{2\alpha_n(\omega)}{1 - \alpha_n(\omega)\alpha(\omega)}\langle f(\omega, \xi(\omega)), J(\xi_{n+1}(\omega), \xi(\omega))\rangle \\
&\le [1 - \frac{2\alpha_n(\omega)(1 - \alpha(\omega))}{1 - \alpha_n(\omega)\alpha}]\|\xi_n(\omega) - \xi(\omega)\|^2 \\
&\quad \frac{2\alpha_n(\omega)(1 - \alpha(\omega))}{1 - \alpha_n(\omega)\alpha(\omega)}[\frac{1}{1 - \alpha(\omega)}\langle f(\omega, \xi(\omega)) - \xi(\omega), J(\xi_{n+1}(\omega) - \xi(\omega))\rangle \\
&\quad + \frac{\alpha_n(\omega)}{2(1 - \alpha(\omega))M_4}],
\end{aligned} \tag{3.11}$$

where M5 is an appropriate constant such that $M_5 \ge \sup_{n\ge 1}\{\|\xi_n(\omega) - \xi(\omega)\|^2\}$. Substi- tuting $j_n = \frac{2\alpha_n(\omega)(1-\alpha(\omega))}{1-\alpha_n(\omega)\alpha(\omega)}$ and

$$t_n = \frac{1}{1 - \alpha(\omega)}\langle f(\omega, \xi(\omega)) - \xi(\omega), J(\xi_{n+1}(\omega) - \xi(\omega))\rangle + \frac{\alpha_n(\omega)}{2(1 - \alpha(\omega))}M_5. \tag{3.12}$$

That is,

$$\|\xi_{n+1}(\omega) - \xi(\omega)\|^2 \leq (1 - j)n)\|\xi_n(\omega) - \xi(\omega)\| + j_n t_n.$$

(3.12)

It implies that, from conditions (i), (ii) and (3.11), that $\lim_{n\to\infty} j_n = 0, \sum_{n=1}^{\infty} j_n = \infty$ and $\limsup_{n\to\infty} t_n \leq 0.$ Apply Lemma 2.3 to (3.12) to conclude $\xi_n(\omega) \to \xi(\omega)$. This completes the proof.

Theorem 3.2*:* *Let T be a random operator defined on a nonempty, closed, convex,i=1 subset K of a separable Hilbert space H. Let $\{T\}_{i}^{r}$= 1: $\Omega \times X \to X$ be a λ_i—strict random pseudo-contraction such that F= $\subset_i^r$= 1 F $(T_i) \neq \emptyset$* Let $\{\mu_i\}\subset$ (0,1) be r real numbers with $\sum_i^r = 1$ $\mu_{i=}$ 1. Given $f \in \prod_k$ and $\xi(\omega) \in K$, for each $\omega \in \Omega$ and sequences $\alpha_n(\omega)$ and $\beta\, n\,(\omega)$, suppose the following control conditions are satisfied

(i) $\lim_{n\to\infty} \alpha_n(\omega) = 0,$

(ii) $\sum_{n=1}^{\infty} \alpha_n(\omega) = \infty,$

(iii) $0 < \beta_n(\omega) < \frac{\lambda}{K^2}$, *for all* $n \geq 0,$

(iv) $\sum_{n=1}^{\infty} |\alpha_{n+1}(\omega) - \alpha_n(\omega)| < \infty, \sum_{n=1}^{\infty} |\beta_{n+1}(\omega) - \beta_n(\omega)| < \infty.$

Then $\{\xi_n(\omega)\}$ *generated by*

$$\xi_{n+1}(\omega) = \alpha_n(\omega) f(\omega, \xi_n(\omega)) + (1 - \alpha_n(\omega)\eta_n(\omega),$$

$$\eta_n(\omega) = \beta_n(\omega) \sum_{i=1}^{r} \mu_i T_i(\omega, \xi(\omega)) + (1 - \beta_n)(\omega)\xi(\omega)$$

Then $\{\xi_n(\omega)\}$ *converges strongly to a random fixed point of* T .

Proof: Define $T\,x = \sum_i^r = 1$ $T_i(\omega, \xi(\omega))$. From Lemma 2.4 and Lemma 2.5, we obtain $T : \Omega \times K \to K$, λ-strict pseudo-contraction with $\lambda = \min\{\lambda_i : 1 \leq i \leq r\}$ and i=1 i=1 $F(T) = F$ $(\sum_i^r = 1\ T_i) = \cap_i^r = 1$ $F(T_i) = F$ From Lemma 2.1, we can conclude the desired
conclusions easily.

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