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MOMENTS OF THE TIME OF RUIN, PENULTIMATE SURPLUS AND DEFICIT AT RUIN UNDER TWO SIDED RISK RENEWAL PROCESS

***[1]Joseph Justin Rebello and [2]Thampi, K. K.**

[1]Department of Statistics, Aquinas College, Kerala-682010, India
[2]Department of Statistics, SNMC, Kerala-683516, India

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ABSTRACT

In this paper we present a concrete method to estimate the moments of time of ruin, surplus immediately before the ruin and deficit at ruin under two way renewal process. Gerber–Shiu discounted penalty function are used to estimate the same. Laplace transform is taken into account while calculating the moment. Furthermore, Lindley distribution is being assumed as the distribution of the claim amount .This approach allows for the experimentation of various distribution forms among the eventual random variables.

INTRODUCTION

The time of ruin, the surplus before ruin and the deficit at ruin etc. have captured tremendous attention of research personalities since its introduction to the literature. There have been remarkable works of the same under classical risk process and renewal risk theory as well. Gerber *et al* (1987) and Defunse and Gerber(1988) focused on densities of time of ruin ,surplus before ruin time, probability of ruin such that claim size distribution is exponential/combination of exponential or a combination of gamma distributions. Dickson *et al*. (1992), Willmort and Lin (1998) Schmidli (1997) discuss properties of the distribution of surplus before the time of ruin, the distribution of the deficit at the time of ruin and their relationship. Di Lorenzo and Tessitore (1996) propose a numerical approximation for calculation of the distribution of the surplus before the time of ruin. Debaen (1990) and Picard and Lefeure (1998, 1999) considered the moment properties of the time of ruin. Later on Gerber and Shiu (1998) study the joint distribution of the time of the ruin, the surplus before and deficit at the time of ruin under discounted penalty function. Willmort and Lin (2000) proposed the method for finding moments of the time of ruin, deficit at ruin, surplus before the ruin using defective renewal equation. Derkic and Willmort (2003) obtained an explicit expression rather a closed form expression for moments.

Yang Xing and Rang Wo (2006) contributed a method for calculating moments under Erlang (n) risk process. All the above said works have been for classical or renewal risk process. Recently many works have been carried out under two sided jumps risk process set up. Perry *et al*. (2002), Jacobson (2005), Xing *et al*.(2008), Zhang *et al*.(2010). Dong *et al*.(2013) contributed to the risk models with two sided jumps. The current paper aims at studying the moments of the time of the ruin, surplus before the ruin and deficit at the ruin under two sided jumps renewal process.to establish the moments we make use of the expected discounted penalty function put forwarded by Gerber and Shiu (1998). The study make use of the distribution of modified waiting time, Rebello and Thampi (working paper, 2016), for the calculation of moments. The number of claims following Poisson distribution, random gain following Erlang (2) and finally we assume that the claim time distribution is following a renewal distribution, Lindleydistribution, with parameter θ.

****Corresponding author: Joseph Justin Rebello***
Department of Statistics, Aquinas College, Kerala-682010, India

Model and Assumptions

Following the idea of Dong and Liu (2013) we consider the surplus process as

$$R(t) = u + pt + \sum_{i=1}^{M(t)} X_i - \sum_{j=1}^{N(t)} Y_j, t \geq 0 \qquad \text{.......................................(1)}$$

Where $u \geq 0$ ≥0,the initial surplus, p>0,the constant premium rate , $\sum_{i=1}^{M(t)} X_i$ Compound Poisson process with intensity λ representing the total random gain (premium income or investment or annuity) until time t. X_i's are independent and identically distributed random variables with common density f and mean μ_x. Here we assume that X follows an Erlangian (2, β) and Laplace transforms of f be $\hat{f}(s) = \int_0^\infty e^{-sx} f(x)dx$ whereas $\sum_{j=1}^{N(t)} Y_j$ is an aggregate claims N(t) is a counting process denoting the number of claims up to time t with interclaim { H_i}.The H_i's are i.i.d random variables with common density g and Laplace transform $\hat{g}(s) = \int_0^\infty e^{-sh} g(h)dh$ .The claim sizes {Y_j} are positive random variables with a common distribution function Q,density q and mean μ_y. Laplace transform $\hat{q}(s) = \int_0^\infty e^{-sy} q(y)dy$ .Here we assume that Y follows Lindley distribution with parameter θ. i.e.

$$q(y) = \frac{\theta^2}{1+\theta}(1+y)e^{-\theta y}, \theta > 0$$

and the Laplace Transform,

$$\hat{q}(\mathrm{s}) = \frac{\theta^2(1+s+\theta)}{(1+\theta)(s+\theta)^2}$$

Also we assume that $\{X_i\},\{Y_j\},\{M(t)\}$ and $\{\mathrm{N(t)}\}$ are mutually independent and $(p+\lambda\mu_x).E(H) > \mu_y$ for ensuring a positive security loading condition.

Let $T = \inf\{t \geq 0, \mathrm{R(t)} < 0\}$ be the ruin time, R (T_) be the surplus immediately before ruin and

$|R(T)|$ be the deficit at ruin, again we define the probability of ruin

$\psi(u) = P[T < \infty / R(0) = 0)$,u≥0.

Again, Let $T = \sum_{i=1}^{n} H_i$ be the time when n^{th} claim occurs,T_0=0.Since ruin only occurs at the epochs where claims occur, then we define the discrete time process $\tilde{R} = \{\tilde{R}_n, n = 0,1,2...\}$ and $\tilde{R}_0 = 0$

Again $\tilde{R}n = R(T_n)$ denotes the surplus immediately after the n^{th} claim.

Now, $\tilde{R}_n = u + pT_n + \sum_{i=1}^{M(\mathrm{T}_n)} X_i - \sum_{k=1}^{n} Y_k$

$= u + p\tilde{T}_n - \sum_{k=1}^{n} Y_k$ where $\tilde{T}_n = T_n + \sum_{i=1}^{M(T_n)} X_i / p$ with $\tilde{T}_0 = 0$ which corresponds to the Sparre Anderson risk model.

$\bar{R}(t) = u + pt - \sum_{i=1}^{\bar{N}(t)} Y_i$ where the initial surplus u and the claim size Y_i are exactly the same as those in model (1).

The counting number process $\bar{N}(t)$ denotes the number of claims up to time t with the modified interclaim times $Z_i = \tilde{T}_i - \tilde{T}_{i-1}$. Clearly Z_i are i.i.d random variables with a common density k.

Due to Rebello and Thampi (working paper, 2016) the modified waiting time distribution is given by $k(t) = a_1 e^{-R_1 t} + a_2 e^{-R_2 t} + a_3 e^{-R_3 t}, t > 0$ where $a_1, a_2, a_3, R_1, R_2 \& R_3$ are properly chosen constants.

Derivation of moments

We have $k(t) = a_1 e^{-R_1 t} + a_2 e^{-R_2 t} + a_3 e^{-R_3 t}, t > 0$ and then $k'(t) = -a_1 R_1 e^{-R_1 t} - a_2 R_2 e^{-R_2 t} - a_3 R_3 e^{-R_3 t}, t > 0$ .Using Gerber-Shiu Penalty function $\Phi_\delta(u) = E[e^{-\delta \tilde{T}} w(R(\tilde{T}\_), |R(\tilde{T})|) I_{(\tilde{T}<\infty)} / R(0) = \text{u}]$ where $\tilde{T}, R(\tilde{T}\_) \& R(\tilde{T})$ are the time of ruin, the deficit at ruin and the surplus before ruin respectively. $I_{()}$ is an indicator function and $w(x, y)$ is a non-negative function of $x_1 > 0 \& x_2 > 0$. We first derive an integro-differential equation for Φ .We consider a delayed renewal process, the elapsed time before the first claim = r and the inter occurrence times after the first claim is greater than r.If the first claim has occurred at time 0 and the ruin has not occurred, the risk model is an ordinary renewal process.
Conditioning on the time and the amount of the first claim, we have,

$$\Phi\ (u) = \text{E}[\text{e}^{-\delta \tilde{T}}\ w(R(\tilde{T}\_), |R(\tilde{T})|).I_{\tilde{T}=r}] + E[\text{e}^{-\delta \tilde{T}}\ w(R(\tilde{T}\_), |R(\tilde{T})|).I_{\tilde{T}>r}]$$

$$\Phi\ (u) = \int_0^\infty e^{-\delta t} k(t) \int_{u+pt}^\infty w(u+pt, x-u-pt) q(\text{x}) dx dt + \int_0^\infty e^{-\delta t} k(t) \int_0^{u+pt} \Phi(u+pt-x) q(\text{x}) dx dt$$

Put s=u+pt, then

$$p\Phi\ (u) = \int_u^\infty e^{-\delta(\frac{s-u}{p})} k(\frac{s-u}{p}) \int_s^\infty w(\text{s}, x-s) q(\text{x}) dx ds + \int_u^\infty e^{-\delta(\frac{s-u}{p})} k(\frac{s-u}{p}) \int_0^s \Phi(\text{s}-x) q(\text{x}) dx ds$$

Consider as $p\Phi(u) = I_1 + I_2 ....(2)$.

Differentiating with respect to u, I_1 becomes

$$I'_1 = \int_u^\infty [e^{-\delta(\frac{s-u}{p})} k'(\frac{s-u}{p}) + k(\frac{s-u}{p}) e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p}] \int_s^\infty w(s, x-s) q(x) dx ds - k(0) \int_u^\infty w(u, x-u) q(x) dx$$

$$I'_1 = \frac{\delta}{p} I_1 + \int_u^\infty e^{-\delta(\frac{s-u}{p})} k'(\frac{s-u}{p}) \int_s^\infty w(s, x-s) q(x) dx ds - k(0) \int_u^\infty w(u, x-u) q(x) dx$$

Differentiating again with respect to u,

$$I''_1 = \frac{\delta}{p} I'_1 + \int_u^\infty [e^{-\delta(\frac{s-u}{p})} k''(\frac{s-u}{p}) \int_s^\infty w(s, x-s) p(x)\, \text{dxds} + k'(\frac{s-u}{p}) e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p}] \int_s^\infty w(s, x-s) q(x) dx ds]$$
$$-k'(0) \int_u^\infty w(u, x-u) p(x) dx - k(0) [\int_u^\infty w(u, x-u) q(x) dx]'$$

$$I''_1 = \frac{\delta^2}{p^2} I_1 + \frac{\delta}{p} \int_u^\infty e^{-\delta(\frac{s-u}{p})} k'(\frac{s-u}{p}) \int_s^\infty w(s, \text{x}-s) q(x) dx ds - \frac{\delta}{p} k(0) \int_u^\infty w(u, x-u) q(x) dx$$
$$+ \int_u^\infty e^{-\delta(\frac{s-u}{p})} k''(\frac{s-u}{p}) \int_s^\infty w(s, x-s) q(x) dx ds + \int_u^\infty k'(\frac{s-u}{p}) e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p} \int_s^\infty w(s, x-s) q(x)\, \text{dxds}$$
$$-k'(0) \int_u^\infty w(u, x-u) q(x) dx - k(0) [\int_u^\infty w(u, x-u) q(x) dx]' ....(*)$$

Similarly for I_2 ,

$$I''_2 = \frac{\delta}{p} I'_2 + \int_u^\infty e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p} k'(\frac{s-u}{p}) \int_0^s \Phi(s-x) q(x) dx ds + \int_u^\infty e^{-\delta(\frac{s-u}{p})} k''(\frac{s-u}{p}) \int_0^s \Phi(s-x) q(x) dx ds$$

$$-k'(0) \int_0^u \Phi(u-x) dx - k(0) [\int_0^u \Phi(u-x) q(x) dx]' ....(**)$$

Using (*) and (**) (2) becomes,

$$p\Phi''(u) = \frac{\delta}{p} I'_1 + \int e^{-\delta(\frac{s-u}{p})} k''(\frac{s-u}{p}) \int_s^\infty w(s, x-s) q(x) dx ds + \int_u^\infty k'(\frac{s-u}{p}) e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p} \int_s^\infty w(s, x-s) q(x) dx ds$$

$$-k'(0) \int_u^\infty w(u, x-u)\, \mathrm{q(x)\, dx} - \mathrm{k(0)} [\int_u^\infty w(u, x-u) q(x) dx]' + \frac{\delta}{p} I'_2$$

$$+\int_u^\infty e^{-\delta(\frac{s-u}{p})} \frac{\delta}{p} k'(\frac{s-u}{p}) \int_0^s \Phi(s-x) q(x) dx ds + \int_u^\infty e^{-\delta(\frac{s-u}{p})} k''(\frac{s-u}{p}) \int_0^s \Phi(s-x) q(x) dx ds$$

$$-k'(0) \int_0^u \Phi(u-x) p(x) dx - k(0) [\int_0^u \Phi(u-x) q(x)\, \mathrm{dx}]'$$

On solving,

$$p\Phi''(u) = \delta\Phi'(u) - \frac{\delta^2}{p}\Phi(u) + \int_u^\infty w(u, x-u) q(x) dx (\frac{\delta}{p} k(0) - 1)$$

$$+\int_0^u \Phi(u-x) q(x) dx (\frac{\delta}{p} k(0) - 1) - k(0) [(\int_u^\infty w(u, x-u) q(x) dx)' + (\int_0^u \Phi(u-x) q(x) dx)']$$

Taking Laplace transform on both sides,

$$p(s^2\hat{\Phi}(s) - s\Phi(0) - \Phi'(0)) = \delta(s\hat{\Phi}(s) - \Phi(0)) - \frac{\delta^2}{p}\hat{\Phi}(s)$$

$$+(\frac{\delta}{p} k(0) - 1)[\hat{w}(s)\, \hat{\mathrm{q}}(\mathrm{s}) + \hat{\Phi}(\mathrm{s})\, \hat{\mathrm{q}}(\mathrm{s})] - \mathrm{k}(0)\, \mathrm{s}[\hat{w}(s)\, \hat{\mathrm{q}}(\mathrm{s}) + \hat{\Phi}(\mathrm{s})\, \hat{\mathrm{q}}(\mathrm{s})]$$

Taking $A_1 = \frac{\delta}{p} k(0) - 1$, we have the Laplace transform of the moment

$$\hat{\Phi}(s) = \frac{(ps-\delta)\Phi(0) + p\Phi'(0) + (A_1 - sk(0))\hat{w}(s)\hat{q}(s)}{ps^2 - \delta s + \frac{\delta^2}{p} + \hat{q}(s)(sk(0) - A_1)}$$

$$\therefore \hat{\Phi}(s) = \frac{(ps-\delta)\Phi(0) + p\Phi'(0) + (A_1 - sk(0))\hat{w}(s)(\frac{\theta^2(1+s+\theta)}{(1+\theta)(s+\theta)^2})}{ps^2 - \delta s + \frac{\delta^2}{p} + (\frac{\theta^2(1+s+\theta)}{(1+\theta)(s+\theta)^2})(sk(0) - A_1)}$$

This paper is constructed to learn the insurance process under two sided jumps risk renewal process. We investigated a reformulated expression for Laplace transform of moments under two sided risk renewal process. The explicit expression for the same is being derived. The application of other feasible distributions to the claim amount may be considered as a scope of further study.

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