



## Full Length Research Article

### DESIGNING OF SKIP-LOT SAMPLING PLAN WITH MULTIPLE REPETITIVE GROUP SAMPLING PLAN AS REFERENCE PLAN THROUGH RELATIVE SLOPES

Suresh, K.K. and \*Lakshmipriya, R.

Department of Statistics, Bharathiar University, Coimbatore-46, Tamilnadu, India

#### ARTICLE INFO

##### Article History:

Received 27<sup>th</sup> August, 2016

Received in revised form

19<sup>th</sup> September, 2016

Accepted 14<sup>th</sup> October, 2016

Published online 30<sup>th</sup> November, 2016

##### Key Words:

Skip lot sampling plans,  
SkSP-R, Multiple Repetitive  
Group Sampling (MRGS) Plan,  
Quality levels,  
Relative slopes.

#### ABSTRACT

This paper explains the selection procedure for Skip lot-sampling plan of type SkSP-R which use Multiple Repetitive Group Sampling (MRGS) plan as reference plan through relative slopes at various points on the OC curve, which describes the degree of steepness of the OC curve.

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#### INTRODUCTION

Industry has accepted Statistical Quality control as a valuable tool for rapid, timely and economical control of process and product output. Such widespread use of sampling techniques for obtaining quality data makes investigations of new quality control systems especially important. Acceptance sampling plan has been widely used in Industries for maintaining high quality level of the product at minimum inspection cost. Acceptance sampling plan are basically divided into two major categories namely; attribute sampling plan and variable sampling plan. The attribute sampling plans are used when the quality characteristic is just classified as good or bad. The variable sampling plans are used when the quality characteristic can be measured on numerical scale. Statistical quality control is simply a statistical method for determining the extent to which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur one exceeding normal expectations. Under skip-lot sampling inspection, samples may be drawn from only a fraction of the submitted lots. Here skipping of lots of good quality protection to the producer is ensured and economy of sample size is achieved.

\*Corresponding author: Lakshmipriya, R.

Department of Statistics, Bharathiar University, Coimbatore-46, Tamilnadu, India

The main purpose of skip-lot sampling is to decrease the frequency of sampling inspection and thus reducing the total inspection costs. Perry (1970, 1973) observed that, from an overall point of view, skip-lotting appears to be good and useful acceptance sampling procedure and might well qualify as a standard system of reduced inspection. The Skip-lot sampling plans have the desired property for reducing the amount of inspection when the quality of submitted products is good. The Skip-lot sampling plan was initially proposed by Dodge (1955) which requires a single determination or analysis to ascertain the lot acceptability or non-acceptability. Such plans are called Skip-lot sampling plans of type SkSP-1. Perry (1973) has studied the properties of SkSP-2 plan with single sampling plan as the reference plan and also provided operating characteristics for SkSP-2 plan with some selected parameters. Balamurali and Subramani (2012) developed Optimal designing of Skip-lot Sampling plan of type SkSP-2 with Double Sampling plan as the reference plan. Vijayaraghavan (2000) introduced Design and evaluation of skip-lot sampling plans of type SkSP-3 plans and the operating characteristics functions are derived using the Markov chain approach. Balamurali and Chi-Hyuck Jun (2010) developed SkSP-V plan based on the principles of CSP-V plan. Further Kavithamani (2014) has studied SkSP-V with various attribute reference plans towards acceptable and limiting quality levels of inspection lots. Recently, a new type of skip-lot sampling

plan called SkSP-R was developed by Balamurali et.al (2014) based on the principle of continuous sampling procedure and resampling scheme for the quality inspection of continuous flow of bulk products

**Review of Skip-lot sampling plan of type SkP-R**

The SkSP –R plan uses the reference plan which is similar to the SkSP-2 plan of Perry (1973). Balamurali, S., Aslam, M. & Jun, C.-H. (2014), was developed new system of skip-lot sampling plan called SkSP-R based on the principle of continuous sampling procedure and resampling scheme for the quality inspection of continuous flow of bulk products.

**Operating procedure for SkSP-R plan**

The operating procedure of SkSP-R is as follows;

- Begin the procedure with normal inspection by applying specified reference plan. During normal inspection, lots are inspected one by one in order of being submitted.
- When i consecutive lots are accepted based on the reference plan under normal inspection, discontinue the normal inspection and switch to skipping inspection
- During the skipping inspection, inspect only a fraction f of lots selected at random by applying reference plan. The skipping inspection is continued until a sampled lot is rejected.
- When a lot is rejected, on skipping inspection, switch to normal inspection.
- During re-inspection procedure, perform the inspection using the reference plan. If the lot is accepted, then continue the skipping inspection. On non-acceptance of the lot, re-inspection is done m times and the lot is rejected if it has been accepted on (m-1)st resubmission.
- If a lot is rejected on the re-inspection scheme, then we immediately revert to normal inspection.
- Replace or correct all non-conforming units found with conforming units in the rejected lots.

**The operating characteristic function of SKSP-R plan**

The operating characteristic function of SKSP-R plan is given

$$P_a(p) = \frac{fP + (1-f)P^i + fp^k(P^i - P)(1-Q^m)}{f(1-p^i)[1 - P^k(1-Q^m) + P^i(1 + fPQ^k)]} \tag{2.2.1}$$

Where p is the probability of acceptance of reference plan.

**Review of Multiple Repetitive Group Sampling (MRGS) plan**

The concept of repetitive group sampling (RGS) plans was introduced by Sherman in which acceptance or rejection of a lot is based on the repeated sample results of the same lot is based on the repeated sample result of the same lot. Recently, Shankar and Mohapatra and Joseph (1993) have proposed a new repetitive group sampling plan as an extension of conditional repetitive group sampling plan in which acceptance or rejection of a lot on the basis of repeated sample is dependent on the outcome of the single sampling inspection under RGS inspection system of the immediately preceding

lots. MRGS is an extension of CRGS plan in which acceptance or rejection of a lot on the basis of repeated sample results is dependent on the outcome of inspection under a RGS inspection system of the preceding lots. For convenience, the proposed plan will be designated as Multiple Repetitive group sampling plan. Second, an attempt has also been made to model and analyses the dynamics of the proposed inspection through GERT (Graphical Evaluation and Review Technique) approach which has been successfully used by several authors for studying a few types of quality control plans. A brief account of researchers in quality control through GERT methods have been given by Shankar [1988]. Suresh and Saminathan (2007) have given a procedure to define Multiple Repetitive Group Sampling plan indexed with MAPD and MAAOQ. Suresh and Kaviyarasu (2011) have studied QSS -1 with Multiple RGS plan as reference plan indexed with AQL, LQL, IQL and its Operating Ratio. Poisson unity values have been tabulated to facilitate the Operation and construction of the plan. Illustration is also provided for selection of plan parameters.

**Operating Procedure**

Draw a random sample of size n and determined the number of defectives (d) found therein.

- Accept the lot , if  $d \leq c_1$
- Reject the lot, if  $d > c_2$
- If  $c_1 < d < c_2$ , repeat the step(1) and (2) provided i successive previous lots are accepted under RGS inspection system, otherwise reject the lot.

The MRGS plans are characterized by four parameters  $n, c_1, c_2$  and acceptance criterion i. here it may be noted that when  $c_1 = c_2$ , the resulting plan is single sampling plan. Also for  $i=0$  one can have the RGS plan of Sherman (1965). It may be noted that the conditions of the application of the proposed plan is same as Sherman RGS plan.

**Operating Characteristic function**

The Operating Characteristic function  $P_a(p)$  of Multiple Repetitive Group Sampling plan is derived by Shankar and Joseph (1993) using Poisson model as

$$P_a(P) = \frac{P_a(1 - P_c)^i}{(1 - P_c)^i - P_c P_a^i} \tag{3.2.1}$$

Where  $P_a = \sum_{r=0}^{c_1} \frac{e^{-np} (np)^x}{x!}$  and

$$P_r = 1 - \sum_{r=0}^{c_2} \frac{e^{-np} (np)^x}{x!}$$

$$P_c = \sum_{r=0}^{c_2} \frac{e^{-np} (np)^x}{x!} - \sum_{r=0}^{c_1} \frac{e^{-np} (np)^x}{x!}$$

And  $h = - \frac{p}{P_a(p)} \frac{dP_a(p)}{dp}$

**Designing plans for given AQL and LQL**

In this paper, three incoming quality levels, namely, Acceptable Quality Level (AQL), Limiting Quality Level

(LQL), and Indifference Quality Level (IQL)) are considered along with their corresponding relative slopes on the OC curve for selection of SkSP-R plans. AQL denoted by  $p_1$ , is the maximum percentage or proportion of variant units in a lot or batch that, for the purposes of acceptance sampling, can be considered as a satisfactory process average. In this paper, AQL is considered as the fraction defective with a 0.95 probability of acceptance of lots. LQL, denoted by  $p_2$ , is the percentage of proportion of variant units in a batch or lot that, for the purposes of acceptance to be restricted to a specified low value, usually 0.10. IQL, denoted by  $p_0$ , is the percentage of variant units in a lot or batch that, for the purposes of acceptance sampling, the probability of acceptance to be restricted to a specific value, namely 0.50. The point  $(p_0, 0.5)$  on the OC curve is referred to as point of control. Fixing the AQL gives adequate protection to the producer and the interests of the consumer are protected by fixing LQL. Fixing IQL gives required protection not only to the consumer but also to the producer.

through  $(p_1, h_1)$  and  $(p_2, h_2)$  involving incentive and filter effects. Vijayaraghavan (1990) has constructed tables and has provided selection procedures for SkSP-2 plans with single sampling plan as reference plan. Table 1 is used to design Skip-lot sampling plan of type SkSP-R with MRGS as reference plan for given  $p_1, p_2, \alpha$  and  $\beta$ . To design a plan for given  $(p_1, 1-\alpha)$  and  $(p_2, \beta)$ , first calculate the operating ratio  $p_2/p_1$ . Then find the value in Table.1 under the column for appropriate  $\alpha$  and  $\beta$ , which is closest to the desired ratio. The  $i_{(MRGS)}$ ,  $f, k, m_{(SkSP)}$  and  $i_{(SkSP)}$  values corresponding to the  $p_2/p_1$  value found in Table 1, can be used to obtain the value of  $np_1$ , where  $P_a(p_1) = 1-\alpha$ . The sample size is determined by dividing  $np_1$  by  $p_1$ .

**Designing plans for given  $(p_1, h_1)$**

Designing plans for given values of  $p_1$  and  $h_1$  use Table 1 for designing the parameters of SkSPMRGS –R.

**Table 1. Relative slopes for SkSP-R with MRGS as reference plan**

$c_1$	$c_2$	$f$	$i_{(SkSP)}$	$k$	$m_{(SkSP)}$	$i_{(MRGS)}$	$np_1$	$h_1$	$np_0$	$h_0$	$np_2$	$h_2$	
1	2	1/2	1	1	1	1	0.9205	0.7364	2.211	2.1115	4.0085	3.9985	
		3					1.3041	0.9465	2.4521	2.3546	4.048	3.9789	
		4					1.6703	1.1025	2.7181	2.6585	4.1126	4.0125	
		5					2.022	1.1125	2.9959	5.8965	4.2098	4.1587	
		6					2.3663	1.2535	3.2823	3.1589	4.3454	4.2358	
1	8	1/3	1	1	1	1	3.1432	2.0456	3.9995	3.8896	4.9477	4.7520	
		2					3.689	2.2356	4.823	4.7652	6.2015	6.0598	
		3					4.1679	3.9563	5.6333	5.5599	7.4893	7.3568	
		4					4.5932	3.9985	6.4607	6.3256	8.7899	8.6589	
		5					4.9658	4.3562	7.3123	7.1256	10.086	9.3258	
1	2	1/2	1	1	1	1	0.9246	0.7562	2.211	2.0115	4.0085	3.8965	
		1/3					1.056	0.9662	2.4696	2.3456	4.4323	3.0085	
		1/5					1.2339	1.1235	2.8457	2.7658	4.9872	4.7589	
		2/5					0.9897	0.8756	2.3506	2.2356	4.2395	4.1256	
		2/7					1.099	1.002	2.5789	2.4568	4.5978	4.3256	
		1/5					1.2339	1.1235	2.8457	2.7562	4.9872	4.7852	
		2					1.1962	1.0965	2.3016	2.1562	3.6653	3.5689	
1	2	1/4	1	1	1	1	1.1638	1.0536	2.0794	2.0005	3.3529	3.2547	
							3	1.1434	0.9965	1.9512	1.8965	3.2745	3.1852
							4	1.1217	0.9452	1.8671	1.7562	3.2579	3.0056
							5	1.1507	1.0365	2.6757	2.5695	4.7426	4.5623
							2	1.144	1.0465	2.6077	2.5632	4.7161	4.6568
1	2	2/3	1	1	1	1	1.1431	1.0332	2.5884	2.2458	4.7147	4.5216	
							3	1.1282	1.0025	2.5827	2.2035	4.7146	4.5016
							4	1.1231	1.0002	2.5809	2.2001	4.7146	4.5016
							5	0.8393	0.7856	2.043	1.9562	3.7206	3.6589
							2	0.8638	0.7562	2.16	1.9456	3.7777	3.5689
1	2	1/5	1	1	1	1	0.8597	0.7362	2.2376	2.1965	3.8271	3.5856	
							3	0.8661	0.7265	2.2908	2.1985	3.8706	3.7423
							4	0.8661	0.7652	2.3281	2.1352	3.9093	3.8546
							5	1.2339	1.0985	2.8457	2.7895	4.9872	4.8756
							2	1.1614	1.0651	2.7802	2.6584	4.9827	4.9822
1	2	1/5	1	1	1	1	1.1201	1.0562	2.7589	2.6587	4.9824	4.9822	
							3	1.088	1.0005	2.7522	2.6589	4.9824	4.9822
							4	1.0608	0.9985	2.7517	2.6485	4.9824	4.9822

The chief features of an OC curve are its location and the relative slope (denoted by h) at that location, which describes the degree of steepness of the OC curve. Hamaker (1950) has made elaborate studies on the slope  $h_0$ , which, along with  $p_0$ , may be used to design any sampling plan. In a similar manner, various other sets of parameters, such as  $(p_1, h_1)$  and  $(p_2, h_2)$  can also be considered for selection of plans. Vedaldi (1986) has studied two principal effects of sampling inspection, which are filter and incentive effects, and has proposed a new criterion based on the AQL and LQL points on the OC curve. Suresh (1993) has presented and constructed tables for the selection of SkSP-2 with Single Sampling Plan as reference plan indexed

For given  $p_1=0.01, h_1= 0.87$ , one can obtain the value of  $np_1$  from Table 1 corresponding to the value of  $h_1= 0.8756$  as  $np_1 = 0.9897$ . The sample size  $n=np_1/p_1 \approx 99$ . The other parameters associated with this  $h_1$  are  $c_1= 1, c_2=2, f=2/5, i_{(SkSP)}=1, i_{(MRGS)}=1, k=1$  and  $m_{(SkSP)}=1$ .

**Designing plans for given  $(p_2, h_2)$**

Designing plans for given values of  $p_2$  and  $h_2$  use Table 1 for designing the parameters of SkSPMRGS-R. For given  $p_2 = 0.02, h_2=9.30$ , from Table.1 under the column headed  $h_2$ , locate the value equal to or just greater than the desired value

$h_2$ . Corresponding to this  $h_2$ , the other values of the parameters are  $c_1=5, c_2=8, f=1/3, i_{(skSP)}=1, k=1, m_{(skSP)}=1$ , and  $i_{(MRGS)}=1$ . From this one can obtain the sample size  $np_2/p_2 \approx 504$ .

**Designing plans for given (p<sub>0</sub>, h<sub>0</sub>)**

Designing plans for given values of  $p_0$  and  $h_0$  use Table 1 for designing the parameters of SkSPMRGS –R. For given  $p_0=0.03, h_0= 2.75$ . From Table 1 under the column headed  $h_0$ , located the value equal to or just greater than the desired  $h_0$  as  $h_0= 2.7562$ . Corresponding to this  $h_0$ , the values of the other parameters are  $c_1=1, c_2=2, f=1/5, i_{(skSP)}=1, k=1, m_{(skSP)}=1$ , and  $i_{(MRGS)}=1$ . From this one can obtain the sample size  $np_0/p_0 \approx 95$ .

**Conversion of Parameters**

One may be interested in converting the given set of parameters into other familiar sets which provide information about the related parameters. For example value of  $p_1= 0.01, h_1= 1.25$  are specified, the other equivalent set of parameters are found using Table.1. Corresponding to  $h_1=1.25$ , one finds tabulated  $h_1= 1.2535$ . The  $np_1$  value associated with this is  $np_1= 2.3663$ . Now the sample size  $n=np_1/p_1 \approx 237$ . The other associated values are  $np_2=4.3454, np_0 = 3.2823, h_2=4.2358$  and  $h_0 =3.1589$ . Therefore  $p_2=np_2/n = 0.018, p_0 =np_0/n = 0.0138$ .

The other set of parameters are,

- $(p_1, p_2) = (0.01, 0.018)$
- $(p_1, p_0) = (0.01, 0.0138)$
- $(p_2, h_2) = (0.018, 4.2358)$
- $(p_0, h_0) = (0.0138, 3.1589)$
- $(p_1, h_1) = (0.01, 1.2535)$

**Construction of Tables**

The expression for the OC function of SkSP-R with MRGS as reference plan is given as

$$P_a(p) = \frac{fP + (1-f)P^i + fp^k(P^i - P)(1-Q^m)}{f(1-p^i)[1 - P^s(1-Q^m) + P^i(1 + fPQ^s)]} \tag{5.1}$$

$$\text{Here } p = \frac{p_a(1-p_c)^i}{(1-p_c)^i - p_c p_a^i} \tag{5.2}$$

Is the OC function for MRGS as reference plan.

For assumed values of  $c_1, c_2, i_{(skSP)}, k, f, i_{(MRGS)}, m_{(skSP)}, P_a(p)$  the equation 5.1 is solved with equation 5.2 for  $np$  using iteration techniques. The relative slopes of the OC curve is given in equation (2.2.1). The value of relative slopes at AQL, LQL and IQL are  $h_1, h_2$  and  $h_0$  values, which are calculated using the  $np_1, np_2, np_0$  values in the formulas,

$$h_1 = \frac{p_1}{p_a(p_1)} \left[ \frac{dp_a(p_1)}{dp} \right]$$

$$h_2 = \frac{p_2}{p_a(p_2)} \left[ \frac{dp_a(p_2)}{dp} \right]$$

$$h_0 = \frac{p_0}{p_a(p_0)} \left[ \frac{dp_a(p_0)}{dp} \right]$$

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