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SEMI- ... -COMPACT SPACE IN A TOPOLOGICAL SPACE

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ABSTRACT

In this paper semi-L-compact, semi-R-compact, semi-L-locally compact, semi-R-locally compact, sequentially semi-L-compact, sequentially semi-R-compact, countably semi-L-compact, countably semi-R-compact are introduced and the relationship between these concepts are studied.

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INTRODUCTION

A.S.Mashhour, M.E Abd El.Monsef and S.N.El-Deeb [6] introduced a new class of semi-open sets in 1982. R.Selvi and M.Priyadarshini introduced a new class of semi-L-open sets in 2016(October). In this paper semi-L-compact, semi-R-compact, semi-L-locally compact, semi-R-locally compact, sequentially semi-L-compact, sequentially semi-R-compact, countably semi-L-compact, countably semi-R-compact are defined and their properties are investigated.

2. Preliminaries

Throughout this paper $f^{-1}(f(A))$ is denoted by A^* and $f(f^{-1}(B))$ is denoted by B^* .

Definition 2.1

Let A be a subset of a topological space (X, τ) . Then A is called semi-open if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed if $\text{int}(\text{cl}(A)) \subseteq A$; [1].

Definition 2.2

Let $f: (X, \tau) \rightarrow (Y, \tau')$ be a function. Then f is semi-continuous if $f^{-1}(B)$ is open in X for every semi-open set B in Y. [1]

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Definition: 2.3

Let $f: (X, \dagger) \rightarrow (Y, \dagger)$ be a function. Then f is semi-open (resp. semi-closed) if $f(A)$ is semi-open (resp. semi-closed) in Y for every semi-open (resp. semi-closed) set A in X . [1]

Definition: 2.4

Let $f: (X, \dagger) \rightarrow Y$ be a function. Then f is

- S-L-Continuous if A^* is open in X for every semi-open set A in X .
- S-M-Continuous if A^* is closed in X for every semi-closed set A in X . [2]

Definition: 2.5

Let $f: X \rightarrow (Y, \dagger)$ be a function. Then f is

- S-R-Continuous if B^* is open in Y for every semi-open set B in Y .
- S-S-Continuous if B^* is closed in Y for every semi-closed set B in Y . [2]

Definition: 2.6

Let $f: (X, \dagger) \rightarrow (Y, \dagger)$ be a function, then f is said to be

- S-irresolute if $f^{-1}(V)$ is semi-open in X , whenever V is semi-open in Y .
- S-resolute if $f(V)$ is semi-open in Y , whenever V is semi-open in X . [4]

Definition: 2.7

Let (X, \dagger) is said to be

- Finitely S-additive if finite union of semi-closed set is semi-closed.
- Countably S-additive if countable union of semi-closed set is semi-closed.
- S-additive if arbitrary union of semi-closed set is semi-closed. [6]

Definition: 2.8

Let (X, \dagger) be a topological space and $x \in X$. Every semi-open set containing x is said to be a S-neighbourhood of x . [3]

Definition: 2.9

Let A be a subset of X . A point $x \in X$ is said to be semi-limit point of A if every semi-neighbourhood of x contains a point of A other than x . [3]

Definition: 2.10

Let A be a subset of a topological space (X, \dagger) , semi-closure of A is defined to be the intersection of all semi-closed sets containing A . It is denoted by $\text{pcl}(A)$. [2]

Definition: 2.11

Let A be a subset of X . A point $x \in X$ is said to be semi-limit point of A if every semi-neighbourhood of x contains a point of A other than x . [5]

Definition: 2.12

A collection \dagger of subsets of X is said to have finite intersection property if for every sub collection $\{C_1, C_2, \dots, C_n\}$ of \dagger the intersection $C_1 \cap C_2 \cap \dots \cap C_n$ is nonempty. [7]

Definition: 2.13

A collection $\{U_\Gamma\}_{\Gamma \in \Delta}$ of semi-open sets in X is said to be semi-open cover of X if $X = \bigcup_{\Gamma \in \Delta} U_\Gamma$. [11]

Definition: 2.14

A topological space (X, \dagger) is said to be semi-compact if every semi-open covering of X contains finite sub collection that also cover X . A subset A of X is said to be semi-compact if every covering of A by semi-open sets in X contains a finite subcover [10]

Definition: 2.15

A subset A of a topological space (X, τ) is said to be countably semi-compact, if every countable semi-open covering of A has a finite subcover.[11]

Example: 2.16

Let (X, τ) be a countably infinite indiscrete topological space. In this space $\{\{x\}/x \in X\}$ is a countable semi-open cover which has no finite subcover. Therefore it is not countably semi-compact.[11]

Definition: 2.17

A subset A of a topological space (X, τ) is said to be sequentially semi-compact if every sequence in A contains a subsequence which semi-converges to some point in A . [9]

Definition: 2.18

A topological space (X, τ) is said to be semi-locally compact if every point of X is contained in a semi-neighbourhood whose semi-closure is semi-compact.[9]

Definition: 2.19

Let $f: (X, \tau) \rightarrow Y$ be a function and A be a subset of a topological space (X, τ) . Then A is called

- S-L-open if $A^* \subseteq cl(int(A^*))$
- S-M-closed if $A^* \supseteq int(cl(A^*))$ [7]

Definition: 2.20

Let $f: X \rightarrow (Y, \tau)$ be a function and B be a subset of a topological space (Y, τ) . Then B is called

- S-R-open if $B^* \subseteq cl(int(B^*))$
- S-S-closed if $B^* \supseteq int(cl(B^*))$ [7]

Example: 2.21

Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$. Let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow Y$ defined by $f(a)=2, f(b)=1, f(c)=3$. Then f is S-L-open and S-M-Closed. [7]

Example: 2.22

Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$. Let $\tau = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Let $g: X \rightarrow (Y, \tau)$ defined by $g(a)=2, g(b)=2, g(c)=3$. Then g is S-R-open and S-S-Closed. [7]

Definition: 2.23

Let $f: (X, \tau) \rightarrow (Y, \tau)$ be a function, then f is said to be

- S-L-irresolute if $f^{-1}(f(A))$ is semi-L-open in X , whenever A is semi-L-open in X .
- S-M-irresolute if $f^{-1}(f(A))$ is semi-M-closed in X , whenever A is semi-M-closed in X .
- S-R-resolute if $f(f^{-1}(B))$ is semi-R-open in Y , whenever B is semi-R-open in Y .
- S-S-resolute if $f(f^{-1}(B))$ is semi-S-closed in Y , whenever B is semi-S-closed in Y . [7]

Definition: 2.24

Let (X, τ) is said to be

- Finitely S-M-additive if finite union of S-M-closed set is S-M-closed.
- Countably S-M-additive if countable union of semi-M-closed set is semi-M-closed.
- S-M-additive if arbitrary union of semi-M-closed set is semi-M-closed. [7]

Definition: 2.25

Let (X, τ) be a topological space and $x \in X$. Every semi-L-open set containing x is said to be a S-L-neighbourhood of x . [7]

Definition: 2.26

Let A be a subset of X . A point $x \in X$ is said to be semi-L-limit point of A if every semi-L-neighbourhood of x contains a point of A other than x . [7]

3. Semi- ... -compact space**Definition: 3.1**

- A collection $\{U_\Gamma\}_{\Gamma \in \Delta}$ of semi-L-open sets in X is said to be semi-L-open cover of X if $X = \bigcup_{\Gamma \in \Delta} U_\Gamma$.
- A collection $\{U_\Gamma\}_{\Gamma \in \Delta}$ of semi-R-open sets in X is said to be semi-R-open cover of X if $X = \bigcup_{\Gamma \in \Delta} U_\Gamma$.

Definition: 3.2

- A topological space (X, τ) is said to be semi-L-compact if every semi-L-open covering of X contains finite sub collection that also cover X . A subset A of X is said to be semi-L-compact if every covering of A by semi-L-open sets in X contains a finite subcover.
- A topological space (X, τ) is said to be semi-R-compact if every semi-R-open covering of X contains finite sub collection that also cover X . A subset A of X is said to be semi-R-compact if every covering of A by semi-R-open sets in X contains a finite subcover.

Theorem: 3.3

A topological space (X, τ) is

- 1) semi-L-compact \Rightarrow compact
- 2) Any finite topological space is semi-L-compact.

Proof:

- Let $\{A_\Gamma\}_{\Gamma \in \Omega}$ be an open cover for X . Then each A_Γ is semi-L-open. Since X is semi-L-compact, this open cover has a finite subcover. Therefore (X, τ) is compact.
- 2) Obvious since every semi-L-open cover is finite.

Example: 3.4

Let (X, τ) be an infinite indiscrete topological space. In this space all subsets are semi-L-open. Obviously it is compact. But $\{x\} \times X \in X$ is a semi-L-open cover which has no finite subcover. So it is not semi-L-compact. Hence compactness need not imply semi-L-compactness.

Theorem: 3.5 A semi-M-closed subset of semi-L-compact space is semi-L-compact.

Proof:

Let A be a semi-M-closed subset of a semi-L-compact space (X, τ) and $\{U_\Gamma\}_{\Gamma \in \Delta}$ be a semi-L-open cover for A , then $\{\{U_\Gamma\}_{\Gamma \in \Delta}, \{X-A\}\}$ is a semi-L-open cover for X . Since X is semi-L-compact, there exists $\Gamma_1, \Gamma_2, \dots, \Gamma_n \in \Delta$ such that $X = U_{\Gamma_1} \cup U_{\Gamma_2} \cup \dots \cup U_{\Gamma_n} \cup (X-A)$ Therefore $A \subseteq U_{\Gamma_1} \cup U_{\Gamma_2} \cup \dots \cup U_{\Gamma_n}$ which proves A is semi-L-compact.

Remark: 3.6

The converse of the above theorem need not be true as seen in the following example(3.7).

Example: 3.7

Let $X = \{a, b, c, \dots\}$ and $Y = \{1, 2, 3, \dots\}$. Let $f: (X, \tau) \rightarrow Y$ defined by $f(a)=1, f(b)=2, f(c)=3$. Let $X = \{a, b, c\}$ $\tau = \{W, \{a\}, X\}$ -open set, closed set- $\{W, X, \{b, c\}\}$. Here $SLO(X) = \{W, X, \{a\}, \{a, b\}, \{a, c\}\}$ is semi-L-compact, $A = \{a, c\}$ is Semi-L-compact but not semi-M-closed

Theorem: 3.8

A topological space (X, τ) is semi-L-compact if and only if for every collection \mathcal{C} of semi-M-closed sets in X having finite intersection property, $\bigcap_{C \in \mathcal{C}} C$ of all elements of \mathcal{C} is non empty.

Proof:

Let (X, τ) be semi-L-compact and \mathcal{C} be a collection of semi-M-closed sets with finite intersection property. Suppose $\bigcap_{C \in \mathcal{C}} C = \emptyset$ then $\bigcup_{C \in \mathcal{C}} (X - C) = X$. Therefore $\{X - C\}_{C \in \mathcal{C}}$ is a semi-L-open cover for X . Then there exists $C_1, C_2, \dots, C_n \in \mathcal{C}$ such that $\bigcup_{i=1}^n (X - C_i) = X$

Therefore $\bigcap_{i=1}^n C_i = \emptyset$ which is a contradiction. Therefore $\bigcap_{C \in \mathcal{C}} C \neq \emptyset$

Conversely assume the hypothesis given in the statement. To prove X is semi-L-compact.

Let $\{U_\alpha\}_{\alpha \in \Delta}$ be a semi-L-open cover for X . then $\bigcup_{\alpha \in \Delta} U_\alpha = X \Rightarrow \bigcap_{\alpha \in \Delta} (X - U_\alpha) = \emptyset$. By hypothesis $\Gamma_1, \Gamma_2, \dots, \Gamma_n$, there exists such that $\bigcap_{i=1}^n (X - U_{\Gamma_i}) = \emptyset$. Therefore $\bigcup_{i=1}^n U_{\Gamma_i} = X$. Therefore X is semi-L-compact.

Corollary: 3.9

Let (X, τ) be a semi-L-compact space and let $C_1 \supseteq C_2 \supseteq \dots \supseteq C_n \supseteq C_{n+1} \dots$ be a nested sequence of nonempty semi-M-closed sets in X . then $\bigcap_{n \in \mathbb{Z}^+} C_n$ is nonempty.

Proof:

Obviously $\{C_n\}_{n \in \mathbb{Z}^+}$ finite intersection property. By theorem (3.8) $\bigcap_{n \in \mathbb{Z}^+} C_n$ is nonempty.

Theorem: 3.10

Let $(X, \tau), (Y, \sigma)$ be two topological space and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijection then

- f is semi-continuous and X is semi-L-compact $\Rightarrow Y$ is compact.
- f is semi-L-irresolute and X is semi-L-compact $\Rightarrow Y$ is semi-L-compact.
- f is continuous and X is semi-L-compact $\Rightarrow Y$ is compact.
- f is strongly irresolute and X is compact $\Rightarrow Y$ is semi-L-compact.
- f is semi-L-open and Y is semi-L-compact $\Rightarrow X$ is compact.
- f is open and Y is semi-L-compact $\Rightarrow X$ is compact.
- f is pre-R-resolute and Y is semi-R-compact $\Rightarrow X$ is semi-R-compact.

Proof:

1) Let $\{U_\alpha\}_{\alpha \in \Delta}$ be a open cover for Y .

Therefore $Y = \bigcup U_\alpha$. Therefore $X = f^{-1}(Y) = \bigcup f^{-1}(U_\alpha)$.

Then $\{f^{-1}(U_\alpha)\}_{\alpha \in \Delta}$ is a semi-L-open cover for X .

Since X is semi-L-compact, there exists $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ such that $X = \bigcup f^{-1}(U_{\Gamma_i})$. Therefore $Y = f(X) = \bigcup (U_{\Gamma_i})$.

Therefore Y is compact.

Proof of (2) to (4) are similar to the above.

5) Let $\{U_\alpha\}_{\alpha \in \Delta}$ be a open cover for X . then $\{f(U_\alpha)\}$ is a semi-L-open cover for Y .

Since Y is semi-L-compact, there exists $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ such that $Y = \bigcup f(U_{\Gamma_i})$

Therefore $X = f^{-1}(Y) = \bigcup_{\alpha \in \Delta} (U_\alpha)$. Therefore X is compact.

Proof of (6) and (7) are similar.

Remark:3.11

From (3) and (6) it follows that “Semi-L- compactness” is a Semi-L- topological property.

Theorem:3.12(Generalisation of Extreme Value theorem)

Let $f: X \rightarrow Y$ be semi-L-continuous where Y is an ordered set in the ordered topology. If X is semi-L-compact then there exists c and d in X such that $f(c) \leq f(x) \leq f(d)$ for every $x \in X$.

Proof

We know that semi-L-continuous image of a semi-L-compact space is compact By theorem(3.10). Therefore $A=f(X)$ is compact. Suppose A has no largest element then $\{(-\infty, a) / a \in A\}$ form an open cover for A and it has a finite subcover.

Therefore $A \subseteq (-\infty, a_1) \cup (-\infty, a_2) \cup \dots \cup (-\infty, a_n)$. Let $a = \max_i a_i$.

Then $A \subseteq (-\infty, a)$ which is a contradiction to the fact that $a \in A$

Therefore A has a largest element M . Similarly it can be proved that it has the smallest element m .

Therefore $\exists c$ and d in X $\exists f(c) = m, f(d) = M$ and $f(c) \leq f(x) \leq f(d) \forall x \in X$.

4. Countably semi - ' ' -compact space

Definition: 4.1

\mathfrak{N} A subset A of a topological space (X, \mathfrak{I}) is said to be countably semi-L-compact, if every countable semi-L-open covering of A has a finite subcover.

\mathfrak{N} A subset A of a topological space (X, \mathfrak{I}) is said to be countably semi-R-compact, if every countable semi-R-open covering of A has a finite subcover.

Example: 4.2

Let (X, \mathfrak{I}) be a countably infinite indiscrete topological space.

In this space $\{\{x\} / x \in X\}$ is a countable semi-L-open cover which has no finite subcover . Therefore it is not countably semi-L-compact.

Remark: 4.3

- Every semi-L-compact space is countably semi-L-compact. It is obvious from the definition.
- Every countably semi-L compact space is countably compact. It follows since open sets are semi-L-open.

Theorem: 4.4

In a countably semi-L-compact topological space, every infinite subset has a semi-L-limit point.

Proof:

Let (X, \mathfrak{I}) be countably semi-L-compact space. Suppose that there exists an infinite subset A which has no semi-L-limit point.

Let $B = \{a_n / n \in \mathbb{N}\}$ be a countable subset of A .

Since B has no semi-L-limit point of B , there exists a semi-L-neighbourhood U_n of a_n such that $B \cap U_n = \{a_n\}$. Now $\{U_n\}$ is a semi-L-open cover for B . Since B^c is semi-L-open, $\{B^c, \{U_n\}_{n \in \mathbb{Z}^+}\}$ is a countable semi-L-open cover for X . But it has no finite sub cover, which is a contradiction, since X is countably semi-L-compact .Therefore every infinite subset of X has a semi-L-limit point.

Corollary: 4.5

In a semi-L-compact topological space every infinite subset has a semi-L-limit point.

Proof:

It follows from the theorem (4.4), since every semi-L-compact space is countably semi-L-compact.

Theorem: 4.6

A semi-M-closed subset of countably semi-L-compact space is countably semi-L-compact.

Proof:

Let X be a semi-L-compact space and B be a semi-M-closed subsets of X

Let $\{A_i / i = 1, 2, 3, \dots, \infty\}$ be a countable semi-L-open cover for B . Then $\{\{A_i\}, X - B\}$

Where $i = 1, 2, 3, \dots, \infty$ is a semi-L-open cover for X . Since X is countably semi-L-compact, there exists $i_1, i_2, i_3, \dots, i_n \ni (X - B) \cup_{k=1}^n A_{i_k} = X$.

Therefore $B = \bigcup_{k=1}^n A_{i_k}$ and this implies B is countably semi-L-compact.

Definition: 4.7

In a topological space (X, τ) a point $x \in X$ is said to be a semi-L-isolated point of A if there exists a semi-L-open set containing x which contains no point of A other than x .

Theorem: 4.8

A topological space (X, τ) is countably semi-L-compact if and only if for every countable collection \mathcal{C} of semi-L-closed sets in X having finite intersection property, $\bigcap_{C \in \mathcal{C}} C$ of all elements of \mathcal{C} is nonempty.

Proof: It is similar to the proof of theorem(3.8).

Corollary: 4.9

X is countably semi-L-compact if and only if every nested sequence of semi-M-closed non empty sets $C_1 \supset C_2 \supset \dots$ has a nonempty intersection.

Proof:

Obviously $\{C_n\}_{n \in \mathbb{Z}^+}$ has finite intersection property. By theorem (4.8) $\bigcap_{n \in \mathbb{Z}^+} C_n$ is nonempty.

5. Sequentially semi- L-compact space**Definition: 5.1**

- A subset A of a topological space (X, τ) is said to be sequentially semi-L-compact if every sequence in A contains a subsequence which semi-L-converges to some point in A .
- A subset A of a topological space (X, τ) is said to be sequentially semi-R-compact if every sequence in A contains a subsequence which semi-R-converges to some point in A .

Theorem: 5.2

Any finite topological space is sequentially semi-L-compact.

Proof:

Let (X, τ) be a finite topological space and $\{x_n\}$ be a sequence in X . In this sequence except finitely many terms all other terms are equal. Hence we get a constant subsequence which semi-L-converges to the same point.

Theorem: 5.3

Any infinite indiscrete topological space is not sequentially semi-L-compact.

Proof:

Let (X, τ) be infinite indiscrete topological space and $\{x_n\}$ be a sequence in X . Let $x \in X$ be arbitrary. Then $U = \{x\}$ is semi-L-open and it contains no point of the sequence except x . Therefore $\{x_n\}$ has no subsequence which semi-L-converges to x . Since x is arbitrary, X is not sequentially semi-L-compact.

Theorem: 5.4

A finite subset A of a topological space (X, \dagger) is sequentially semi-L-compact.

Proof:

Let $\{x_n\}$ be an arbitrary sequence in X . Since A is finite, at least one element of thesequence say x_0 must be repeated infinite number of times. So the constant subsequence x_0, x_0, \dots must semi-L-converges to x_0 .

Remark: 5.5

Sequentially semi-L-compactness implies sequentially compactness, since allopen sets are semi-L-open. But the inverse implication is not true as seen from(5.6).

Example: 5.6

Let (X, \dagger) be an infinite indiscrete space is sequentially compact but notsequentially semi-L-compact.

Theorem: 5.7

Every sequentially semi-L-compact space is countably semi-compact.

Proof:

Let (X, \dagger) be sequentially semi-L-compact. Suppose X is not countably semi-L-compact. Thenthere exists countable pre-open cover $\{U_n\}_{n \in \mathbb{Z}^+}$ which has no finite sub cover .Then $X = \bigcup_{n \in \mathbb{Z}^+} U_n$. Choose $X_1 \in U_1, X_2 \in U_2 - U_1, X_3 \in U_3 - \bigcup_{i=1,2} U_i, \dots, X_n \in U_n - \bigcup_{i=1}^n U_i$. This is possible since $\{U_n\}$ has no finite sub cover. Now $\{x_n\}$ is a sequence in X . Let $x \in X$ bearbitrary .then $x \in U_k$ for some K .By our choice of $\{x_n\}$, $x_i \notin U_k$ for all $i \geq k$. Hence there isno subsequence of $\{x_n\}$ which can semi-L-converge to x . Since x is arbitrary the sequence $\{x_n\}$ has no semi-L-convergent subsequence which is a contradiction. Therefore X is countablysemi-L-compact.

Theorem: 5.8

Let $f: (X, \dagger) \rightarrow (Y, \dagger)$ be a bijection, then

- 1) f is semi-R-resolute and Y is sequentially semi -R-compact $\Rightarrow X$ is sequentiallysemi -R-compact.
- 2) f is semi -L-irresolute and X is sequentially semi -compact $\Rightarrow Y$ is sequentially semi -L-compact.
- 3) f is continuous and X is sequentially semi -L-compact $\Rightarrow Y$ is sequentially semi -L-compact.
- 4) f is strongly semi -L-continous and X is sequentially semi -L-compact $\Rightarrow Y$ is sequentially semi -L-compact.

Proof:

1) Let $\{x_n\}$ be a sequence in X .Then $\{f(x_{nk})\}$ is a sequence in Y . It has asemi -R-convergent subsequence $\{f(x_{nk})\}$ such that $\{f(x_{nk})\} \xrightarrow{pre} y_0$ in Y . Then there exists $x_0 \in X$ such that $f(x_0) = y_0$. Let U be semi -R -open set containing x_0 then $f(U)$ is a semi -R-open setcontaining y_0 .Then there exists N such that $f \in f(U)$ for all $k \geq N$.

Therefore $f^{-1} \circ f(x_{nk}) \in f^{-1} \circ f(U)$. Therefore $x_{nk} \in U$ for all $k \geq N$.

This proves that X is sequentiallysemi -R-compact. Proof for (2) to (4) is similar to the above.

Remark: 5.9

From theorem (5.8), (1) and (2) it follows that ‘‘Sequentially compactness’’ is a semi - ... -topological property.

6.Semi - ... -locally compact space**Definition: 6.1**

A topological space (X, \dagger) is said to be semi -L-locally compact if every point of X is contained in a semi -L-neighbourhood whose semi -L-closure is semi -L-compact.

Theorem: 6.2

Any semi -L-compact space is semi -L-locally compact.

Proof:

Let (X, \mathcal{L}) be semi -L-compact, Let $x \in X$ then X is semi -L-neighbourhood of x and $Scl(X)=X$ which is semi -L-compact.

Remark: 6.3

The converse need not be true as seen in the following example(6.4)

Example: 6.4

Let (X, \mathcal{L}) be an infinite indiscrete topological space. it is not semi -L-compact. But for every $x \in X$, $\{x\}$ is a semi -L-neighbourhood and $\{\bar{x}\} = \{x\}$ is semi -L-compact. Therefore it is semi -L-locally compact.

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