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(r^*g^*)* CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce (r^*g^*)* continuous functions and study some of the properties.

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INTRODUCTION

N Levine [8] introduced the class of semi continuous functions using semi open sets. Balachandran et al in [2] introduced the concept of generalized continuous maps in a topological space. Many authors introduced several generalized closed sets and generalized continuous maps. The Authors [11] have already introduced (r^*g^*)* closed sets and investigated some of their properties. In this paper we introduce a new class of maps called (r^*g^*)* continuous maps. Also (r^*g^*)* irresolute map is introduced.

Priliminaries

Definition: 2.1 A subset A of a space X is called

1. A generalized closed (g closed) [7] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
2. A Regular generalized closed (rg-closed) [15] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
3. A generalized pre regular closed (gpr closed)[6] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
4. A g^* closed [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.
5. A regular weakly generalized semi closed (rwg closed) [12] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
6. A g^{**} closed [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open.

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7. A $g\#$ closed [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg open.
8. A generalized semi-preclosed star closed ($(gsp)^*$ closed)[13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp open.
9. A gp^* closed [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gp -open.
10. A regular[^] generalized closed ($r^{\wedge}g$ closed)[16] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
11. A regular generalized b-closed (rgb closed) [9] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
12. A $(r^*g^*)^*$ closed set [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is r^*g^* - open.

Definition 2.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- (i) rg - continuous [15] if $f^{-1}(V)$ is rg closed in (X, τ) for every closed set V of (Y, σ) .
- (ii) gpr - continuous [6] if $f^{-1}(V)$ is gpr closed in (X, τ) for every closed set V of (Y, σ) .
- (iii) rwg continuous [12] if $f^{-1}(V)$ is rwg closed in (X, τ) for every closed set V of (Y, σ) .
- (iv) $r^{\wedge}g$ continuous [16] if $f^{-1}(V)$ is $r^{\wedge}g$ closed in (X, τ) for every closed set V of (Y, σ) .
- (v) rgb continuous [9] if $f^{-1}(V)$ is rgb closed in (X, τ) for every closed set V of (Y, σ) .
- (vi) g^* continuous [18] if $f^{-1}(V)$ is g^* closed in (X, τ) for every closed set V of (Y, σ) .
- (vii) g^{**} continuous [14] if $f^{-1}(V)$ is g^{**} closed in (X, τ) for every closed set V of (Y, σ) .
- (viii) $g\#$ continuous [17] if $f^{-1}(V)$ is $g\#$ closed in (X, τ) for every closed set V of (Y, σ) .
- (ix) $(gsp)^*$ continuous [13] if $f^{-1}(V)$ is $(gsp)^*$ closed in (X, τ) for every closed set V of (Y, σ) .
- (x) $(gp)^*$ - continuous [7] if $f^{-1}(V)$ is $(gp)^*$ closed in (X, τ) for every closed set V of (Y, σ) .

3. $(r^*g^*)^*$ -CONTINUOUS AND IRRESOLUTE MAPS

Definition 3.1: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $(r^*g^*)^*$ -continuous if the inverse image of every closed set in (Y, σ) is $(r^*g^*)^*$ -closed in (X, τ) .

Theorem 3.2:

Every continuous map is $(r^*g^*)^*$ -continuous.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map. Let F be a closed set in (Y, σ) . Then $f^{-1}(F)$ is closed in (X, τ) . Since every closed set is $(r^*g^*)^*$ -closed $\Rightarrow f^{-1}(F)$ is $(r^*g^*)^*$ -closed set. Therefore f is $(r^*g^*)^*$ -continuous.

The converse need not be true as seen from the following example.

Example 3.3

Let $X = \{a, b, c\}$ $\tau = \{ \phi, X, \{c\}, \{b, c\} \}$ Closed set of $X = \{ \phi, X, \{a, b\}, \{a\} \}$

$(r^*g^*)^*$ closed sets are $\{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$

Let $Y = \{a, b, c\}$, $\sigma = \{ \phi, Y, \{b\} \}$ Closed set of $Y = \{ \phi, Y, \{a, c\} \}$

Let f be the identity mapping. $\{a, c\}$ is closed in (Y, σ) . Now $f^{-1}\{a, c\} = \{a, c\}$ is not closed in (X, τ) Hence f is not continuous. But $\{a, c\}$ is $(r^*g^*)^*$ closed set. Therefore f is $(r^*g^*)^*$ continuous.

Theorem:3.4

Every $(r^*g^*)^*$ -continuous map is rg -continuous.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $(r^*g^*)^*$ -continuous map. Let V be a closed set in (Y, σ) .

Since f is $(r^*g^*)^*$ -continuous, $f^{-1}(V)$ is $(r^*g^*)^*$ -closed in (X, τ) . By proposition 3.7[11]

$f^{-1}(V)$ is rg -closed in (X, τ) . Therefore, f is rg -continuous.

The converse need not be true as seen from the following example.

Example:3.5

Let $X = \{a, b, c\}$ $\tau = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$ Closed set of $X = \{ \phi, X, \{b, c\}, \{a, c\}, \{c\} \}$

$(r^*g^*)^*$ closed sets are $\{ \phi, X, \{c\}, \{b, c\}, \{a, c\} \}$

Let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{c\}\}$, Closed set of $Y = \{\emptyset, Y, \{a, b\}\}$

Let f be defined as $f(a)=b$, $f(b)=a$, $f(c)=c$. Now $\{a, b\}$ is closed in Y .

Now $f^{-1}\{a, b\} = \{a, b\}$ is rg closed in (X, τ) Hence f is rg continuous. But $\{a, b\}$ is not $(r^*g^*)^*$ closed set. Therefore f is not $(r^*g^*)^*$ continuous.

Theorem 3.6

Every $(r^*g^*)^*$ -continuous map is gpr -continuous.

Proof : Follows from proposition 3.9 [11].

The converse need not be true as seen from the following example.

Example:3.7

Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ Closed set of $X = \{\emptyset, X, \{b, c\}\}$

$(r^*g^*)^*$ closed sets are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$

Let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{b, c\}\}$ Closed set of $Y = \{\emptyset, Y, \{a\}\}$

$f: (X, \tau) \rightarrow (Y, \sigma)$ be the map and Let $f(a)=a$, $f(b)=c$, $f(c)=b$.

Now $\{a\}$ is closed in Y . But $f^{-1}\{a\} = \{a\}$ is gpr closed but not $(r^*g^*)^*$ closed in (X, τ) . Hence f is gpr continuous but not $(r^*g^*)^*$ continuous.

Theorem 3.8

Every $(r^*g^*)^*$ -continuous map is rwg -continuous

Proof: Follows from proposition 3.11 [11].

The converse need not be true as seen from the following example.

Example: 3.9

Let $X=Y=\{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, $(r^*g^*)^*$ closed sets are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a, c\}\}$. Closed set of $Y = \{\emptyset, Y, \{b\}\}$ $f: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a)=b$, $f(b)=c$, $f(c)=a$. Then $f^{-1}\{b\} = \{a\}$ is not $(r^*g^*)^*$ -closed in (X, τ) . But $\{a\}$ is rwg -closed. Hence f is rwg continuous but not $(r^*g^*)^*$ continuous.

Theorem 3.10

Every $(r^*g^*)^*$ -continuous map is $r^{\wedge}g$ -continuous

Proof : Follows from proposition 3.21 [11].

The converse need not be true as seen from the following example.

Example:3.11

Let $X=Y=\{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $(r^*g^*)^*$ closed sets of X are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$. $\sigma = \{\emptyset, Y, \{c\}\}$. σ closed sets are $\{\emptyset, Y, \{a, b\}\}$.

$f: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a)=b$, $f(b)=a$, $f(c)=c$. Now $\{a, b\}$ is closed in Y . Here

$f^{-1}\{a, b\} = \{a, b\}$ is $r^{\wedge}g$ closed But not $(r^*g^*)^*$ -closed in (X, τ) . Hence f is not $(r^*g^*)^*$ continuous

Theorem 3.12

Every $(r^*g^*)^*$ -continuous map is rgb -continuous

Proof : Follows from proposition 3.23 [11].

The converse need not be true as seen from the following example.

Example: 3.13

Let $X=Y=\{a,b,c\}$, $\tau = \{ \phi, X, \{a\}, \{b\}, \{a,b\} \}$, $(r^*g^*)^*$ closed sets are $\phi, X, \{c\}, \{b,c\}, \{a,c\}$ $\sigma = \{ \phi, Y, \{a,c\} \}$. σ closed sets are $\phi, Y, \{b\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a)=c, f(b)=b, f(c)=a$. Now $\{b\}$ is closed in Y . Here $f^{-1}\{b\}=\{b\}$ is rgb closed But not $(r^*g^*)^*$ -closed in (X, τ) . Hence f is not $(r^*g^*)^*$ continuous.

Theorem 3.14:

Every g^* -continuous map is $(r^*g^*)^*$ -continuous

Proof : Follows from proposition 3.5 [11].

The converse need not be true as seen from the following example.

Example:3.15 Let $X=Y=\{a,b,c\}$, $\tau=\{ \phi, X, \{a\} \}$, $(r^*g^*)^*$ closed sets are $\{ \phi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\} \}$. g^* closed sets are $\phi, X, \{b,c\}$. $\sigma = \{ \phi, Y, \{b\} \}$. σ closed sets are $\phi, Y, \{a,c\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a)=c, f(b)=b, f(c)=a$. Now $f^{-1}\{a,c\}=\{a,c\}$ Which is $(r^*g^*)^*$ closed but not g^* closed. Hence f is $(r^*g^*)^*$ continuous but not g^* continuous.

Theorem :3.16

Every g^{**} -continuous map is $(r^*g^*)^*$ -continuous.

Proof : Follows from proposition 3.13 [11].

The converse need not be true as seen from the following example.

Example:3.17

Let $X=Y=\{a,b,c\}$, $\tau = \{ \phi, X, \{a\}, \{a,c\} \}$, $(r^*g^*)^*$ closed sets = $\{ \phi, X, \{b\}, \{a,b\}, \{b,c\} \}$, $(r^*g^*)^*$ closed sets $\{ \phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,c\} \}$. $\sigma = \{ \phi, Y, \{b\} \}$. σ closed sets are $\phi, Y, \{a,c\}$, $f: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a)=c, f(b)=b, f(c)=a$. Now $\{a,c\}$ is closed in Y . Here $f^{-1}\{a,c\}=\{a,c\}$ is $(r^*g^*)^*$ closed But not g^{**} -closed in (X, τ) . Hence f is $(r^*g^*)^*$ -continuous but not g^{**} continuous.

Theorem 3.18

Every $g\#$ -continuous map is $(r^*g^*)^*$ continuous.

Proof : Follows from proposition 3.15 [11].

The converse need not be true as seen from the following example.

Example 3.19

Let $X=Y=\{a,b,c\}$, $\tau = \{ \phi, X, \{a\} \}$, $(r^*g^*)^*$ closed sets are $\{ \phi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\} \}$ \mathcal{A} - closed sets $\{ \phi, X, \{b,c\}, \{c\}, \{b\} \}$ \mathcal{A} g - closed sets $\{ \phi, X, \{b\}, \{c\}, \{ab\}, \{bc\}, \{ac\} \}$, \mathcal{A} g - open sets $\{ \phi, X, \{a,c\}, \{a,b\}, \{c\}, \{a\}, \{b\} \}$, $\sigma = \{ \phi, Y, \{a\}, \{b\}, \{a,b\} \}$, σ closed sets are $\phi, Y, \{b,c\}, \{a,c\}, \{c\}$ $g: (X, \tau) \rightarrow (Y, \sigma)$ is defined by $g(a)=b, g(b)=c, g(c)=a$.

$g^{-1}\{b,c\}=\{a,b\}$ is $(r^*g^*)^*$ closed. $g^{-1}\{a,c\}=\{c,b\}$ is $(r^*g^*)^*$ closed. $g^{-1}\{c\}=\{b\}$ is $(r^*g^*)^*$ closed g is $(r^*g^*)^*$ continuous. Now $g^{-1}\{b,c\}=\{a,b\}$, Which is not $g\#$ closed. Hence g is not $g\#$ continuous.

Theorem: 3.20

Every $(gsp)^*$ -continuous map is $(r^*g^*)^*$ -continuous.

Proof : Follows from proposition 3.17 [11].

The converse need not be true as seen from the following example.

Example: 3.21

Let $X=\{a,b,c\}$ $\tau = \{ \phi, X, \{a\} \}$, $(r^*g^*)^*$ closed sets are $\{ \phi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\} \}$. gsp open sets are $\{ \phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\} \}$
Let $Y=\{a,b,c\}$ $\sigma = \{ \phi, Y, \{b\} \}$ σ closed sets are $\phi, Y, \{a,c\}$

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a)=b, f(b)=a, f(c)=c$. Now $\{a, c\}$ is closed in Y . But $f^{-1}\{a, c\}=\{a, c\}$ is $(r^*g^*)^*$ -closed but not $(gsp)^*$ -closed. Hence f is $(r^*g^*)^*$ -continuous but not $(gsp)^*$ -continuous

Theorem 3.22

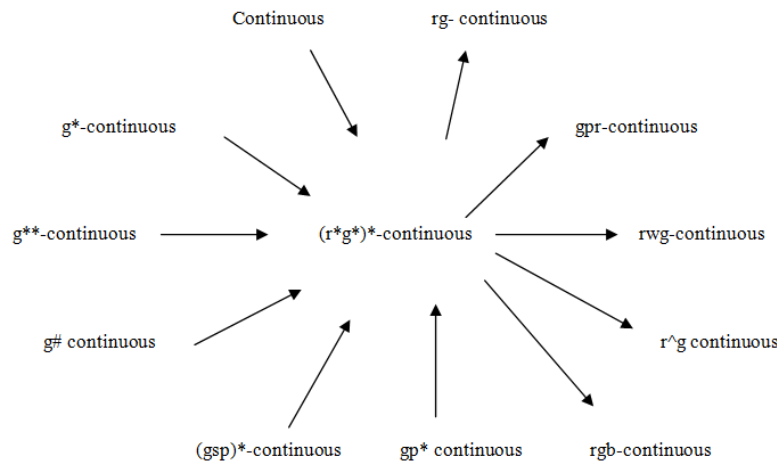
Every $(gp)^*$ -continuous map is $(r^*g^*)^*$ -continuous.

Proof : Follows proposition from 3.19 [11].

The converse need not be true as seen from the following example.

Example 3.23:

Let $X=\{a, b, c\}$ $\tau = \{ \phi, X, \{c\}, \{b, c\} \}$,
 Let $Y=\{a, b, c\}$ $\sigma = \{ \phi, Y, \{c\} \}$ σ closed sets are $\phi, Y, \{a, b\}$
 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a)=a, f(b)=c, f(c)=b$. Now $\{a, b\}$ is closed in Y . But $f^{-1}\{a, b\}=\{a, c\}$ is $(r^*g^*)^*$ -closed but not $(gp)^*$ -closed. Hence f is $(r^*g^*)^*$ -continuous but not $(gp)^*$ -continuous. Thus we have the following Diagram.



where $A \rightarrow B$ represents A implies B and B need not imply A .

Note : $(r^*g^*)^*$ -continuous is independent of pre continuous, semi continuous, semipre continuous, wg continuous, α continuous, sg continuous, and gs continuous [11].

Proposition: 3.24

Composition of two $(r^*g^*)^*$ continuous functions need not be $(r^*g^*)^*$ -continuous. The following example supports the above proposition.

Example 3.25: Let $X=\{a, b, c\}$ $\tau = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$, $(r^*g^*)^*$ closed sets are $\{ \phi, X, \{c\}, \{b, c\}, \{a, c\} \}$ $Y=\{a, b, c\}$, $\sigma = \{ \phi, Y, \{a\} \}$, $(r^*g^*)^*$ closed sets are $\{ \phi, Y, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\} \}$. $Z=\{a, b, c\}$ $\eta = \{ \phi, Z, \{a, c\} \}$

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by $f(a) = b, f(b) = a, f(c) = c$. Define $g : (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a)=c, g(b)=b, g(c)=a$. Here $f^{-1}\{b, c\}=\{a, c\}$ which is $(r^*g^*)^*$ closed and $g^{-1}\{b\}=\{b\}$ which is $(r^*g^*)^*$ closed and hence they are $(r^*g^*)^*$ continuous. But $(gof)^{-1}\{b\}=f^{-1}\{g^{-1}\{b\}\}=f^{-1}\{b\}=\{a\}$ which is not $(r^*g^*)^*$ closed.

Hence (gof) is not $(r^*g^*)^*$ -continuous.

Definition 3.26

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a $(r^*g^*)^*$ -irresolute map if $f^{-1}(V)$ is a $(r^*g^*)^*$ -closed set in (X, τ) for every $(r^*g^*)^*$ -closed set V of (Y, σ) .

Example 3.27

Let $X=\{a, b, c\}$, $\tau = \{ \phi, X, \{a\} \}$ Closed set = $\{ \phi, X, \{b, c\} \}$

$(r^*g^*)^*$ closed set are $\{ \phi, X, \{b\}, \{a,b\}, \{c\}, \{b,c\}, \{a,c\} \}$
 $\sigma = \{ \phi, Y, \{a\}, \{b\}, \{a,b\} \}$ Closed set of $Y = \{ \phi, Y, \{b,c\}, \{a,c\}, \{c\} \}$
 $(r^*g^*)^*$ closed set of Y are $\{ \phi, Y, \{c\}, \{b,c\}, \{a,c\} \}$

Here Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(c)=c, f(b)=a, f(a)=b, f^{-1}(\{c\})=\{c\}, f^{-1}(\{b,c\})=\{a,c\}, f^{-1}(\{a,c\})=\{b,c\}$ which are $(r^*g^*)^*$ closed in (X, τ) . Hence f is an irresolute map.

Theorem 3.28

Every $(r^*g^*)^*$ irresolute map is $(r^*g^*)^*$ -continuous.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $(r^*g^*)^*$ -irresolute map.

Let F be a closed set in (Y, σ) . But every closed set is $(r^*g^*)^*$ closed.

Since f is irresolute map, $\Rightarrow f^{-1}(F)$ is $(r^*g^*)^*$ -closed set in (X, τ)

$\Rightarrow f$ is $(r^*g^*)^*$ -continuous

Therefore, Every $(r^*g^*)^*$ -irresolute map is $(r^*g^*)^*$ -continuous map. The converse need not be true as seen from the following example.

Example 3.29

Let $X=Y=\{a,b,c\}, \tau = \{ \phi, X, \{a\}, \{a,c\} \}$, Closed set = $\{ \phi, X, \{b,c\}, \{b\} \}$ $(r^*g^*)^*$ closed set are of X are $\{ \phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,c\} \}$ $\sigma = \{ \phi, Y, \{a\} \}$ Closed set = $\{ \phi, Y, \{b,c\} \}$ $(r^*g^*)^*$ closed sets of Y are $\{ \phi, Y, \{b\}, \{a,b\}, \{c\}, \{b,c\}, \{a,c\} \}$ Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a, f(b)=c, f(c)=b$. Here $f^{-1}(\{b,c\})=\{c,b\}$ is $(r^*g^*)^*$ closed. Therefore f is $(r^*g^*)^*$ continuous. But $f^{-1}(\{a\})=\{c\}$ is not $(r^*g^*)^*$ closed in (X, τ) . Therefore f is not $(r^*g^*)^*$ irresolute.

Remark 3.30

Every $(r^*g^*)^*$ -irresolute map is rg -continuous, gpr -continuous, rwg -continuous, rgb -continuous, r^*g -continuous.

Theorem 3.31

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two function. Then

- $g \circ f$ is $(r^*g^*)^*$ -continuous if g is continuous and f is $(r^*g^*)^*$ -continuous.
- $g \circ f$ is $(r^*g^*)^*$ -irresolute if both f and g are $(r^*g^*)^*$ -irresolute.
- $g \circ f$ is $(r^*g^*)^*$ -continuous if g is $(r^*g^*)^*$ -continuous and f is $(r^*g^*)^*$ -irresolute.

Proof:

- Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $(r^*g^*)^*$ -continuous and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be continuous. Let F be a closed set in (Z, η) . Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ) . Since f is continuous, $f^{-1}(g^{-1}(F))$ is $(r^*g^*)^*$ -closed in (X, τ) Which $\Rightarrow (g \circ f)^{-1}(F)$ is $(r^*g^*)^*$ -closed. Therefore $g \circ f$ is $(r^*g^*)^*$ -continuous.
- Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $(r^*g^*)^*$ -irresolute map and let $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a $(r^*g^*)^*$ -irresolute map. Let F be a $(r^*g^*)^*$ -closed set in (Z, η) , Since g is $(r^*g^*)^*$ -irresolute map, $g^{-1}(F)$ is $(r^*g^*)^*$ -closed in (Y, σ) , Since f is $(r^*g^*)^*$ -irresolute map, $f^{-1}(g^{-1}(F))$ is $(r^*g^*)^*$ -closed in (X, τ) Which $\Rightarrow (g \circ f)^{-1}(F)$ is $(r^*g^*)^*$ -closed (X, τ) . $\Rightarrow g \circ f$ is $(r^*g^*)^*$ -irresolute map.
- Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $(r^*g^*)^*$ -irresolute and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be $(r^*g^*)^*$ -continuous. Let F be a closed set in (Z, η) . Since g is $(r^*g^*)^*$ -continuous, $g^{-1}(F)$ is $(r^*g^*)^*$ -closed in (Y, σ) . Since f is $(r^*g^*)^*$ -irresolute, $f^{-1}(g^{-1}(F))$ is $(r^*g^*)^*$ -closed in (X, τ) .

$\Rightarrow (g \circ f)^{-1}(F)$ is $(r^*g^*)^*$ -closed. Which $\Rightarrow (g \circ f)$ is $(r^*g^*)^*$ -continuous

REFERENCES

Abd El-Monsef, M.E., El-Deeb, S.N. and Mohamoud, R.A.1983. β open sets and β continuous mappings, *Bull. Fac.Sci. Assiut Univ.*, 12, 77-80.

- Balachandran, K., Sundaram, P. and Maki, H.1991. On generalized continuous maps in topological spaces, *Mem.Fac.Kochi univ.ser.A.Maths.*, 12, 5-13.
- Biswas, N., 1970. On Characterizations of semi-continuous functions, *Atti, Accad. Naz. Lincei Rend. Cl.Fis.Mat.Natur.*, 48(8), 399-402.
- Devi, R., Maki, H. and Balachandran, K. 1993. Semi Generalized closed maps and generalized closed maps, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.*, 14,41-54
- Devi, R., Maki, H. and Balachandran, K. 1998. Generalized α -closed maps and α generalized closed maps, *Indian.J.Pure.Appl.Math*, 29(1),37-49.
- Gnanambal, Y. 1997. On generalized pre regular closed sets in Topological Spaces, *Indian J.Pure App.Maths*, 28, 351-360.
- Jayakumar, P., Mariyappa, K., Sekar, S. 2013. On generalized gp^* closed sets in topological spaces et al, *Int. journal of Math analysis*, Vol 7. no 33,1635-1645.
- Levine, N. 1970. Generalized closed sets in topology, *Rend.Circ.Math.Palermo*,19(2),89-96.
- Mariappa, K. and Sekar, S. 2013. On Regular Generalised b-closed Set, *Int. Journal of Math. Analysis*, vol, no.13,613-624.
- Meenakumari, N. and Indira , T. 2015. On(r^*g^*)* closed sets in topological spaces, *International Journal of Science and Research (IJSR) ISSN (Online): 2319-7064*, Volume 4 Issue 12, December.
- Meenakumari, N. and Indira, T. 2014. r^*g^* closed sets in topological spaces , *Annals of Pure and Applied Mathematics* vol.6, No. 2, 125-132.
- Mugundan, C., Nagaveni, N, 2011. A Weaker form of closed sets, 949-961.
- Palaniappan, N. and Rao, K.C. 1993. Regular generalized closed sets, *Kyungpook Math*.3(2),211.
- Pauline Mary Helen, M. and Kulandai, A. 2014. Therese, (gsp)*-closed sets in topological spaces,IJMTT-Volume 6 February 2014,ISSN NO2231-5373.
- Pauline Mary Helen, M., 2012. Veronica Vijayan, Ponnuthai Selvarani, g^{**} closed sets in Topological Spaces, *IJMA* 3(5), 1-15.
- Savithri, D. and Janaki, C,On Regular^Generalizedclosed sets in Topological spaces,*IJMA- 4(4)2013*,162-169.
- Veerakumar, M.K.R.S. 2000. Between closed sets and g closed sets, *Mem.Fac.Sci.Koch Univ.Ser.A. Math.*, 21, 1-19.
- Veerakumar, M.K.R.S. 2003. $g^\#$ -closed sets in topological spaces, *Mem. Fac.Sci.Kochi Univ Ser.A.,Math.*,24,1-13.
