Available online at http://www.journalijdr.com

International Journal of DEVELOPMENT RESEARCH

ISSN: 2230-9926

International Journal of Development Research
Vol. 05, Issue, 08, pp. 5245-5248, August, 2015

Full Length Research Article

TRIANGULAR DIVISOR CORDIAL LABELING FOR SOME SPECIAL GRAPHS

[1,*] Ponmoni, A., [2]Navaneetha Krishnan, S. and [3]Nagarajan, A.

[1]Department of Mathematics, C.S.I. College of Engineering, Ketti – 643215, Tamilnadu, India
[2,3]Department of Mathematics, V.O.C. College, Tuticorin-628008, Tamilnadu, India

ARTICLE INFO

Article History:
Received 10th May, 2015
Received in revised form
28th June, 2015
Accepted 02nd July, 2015
Published online 31th August, 2015

Key words:
Cordial labeling,
Divisor cordial labeling,
Triangular divisor cordial labeling.

ABSTRACT

Let $G = (V, E)$ be a (p, q)- graph. A Triangular divisor cordial labeling of a graph G with vertex set V is a bijection$f : V \to \{T_1, T_2, T_3, \ldots, T_p\}$ where T_i is the i^{th} Triangular number such that if each edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.If a graph has a Triangular divisor cordial labeling, then it is called Triangular divisor cordial graph. In this paper, we proved the standard graphs such as $C_4^{(t)}$,$K_{1,n}^+$,$F_m \oplus K_{1,n}^+$,$C_n @ K_{1,m}$,$P_{4,n}$,$B_{n,n}$are Triangular divisor cordial graphs.

INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiples edges, for terms not defined here, we refer to Harary (1972). In a labeling of a particular type, the vertices are assigned values from a given set, the edges have a prescribed induced labeling must satisfy certain properties. An excellent reference on this subject is the survey by Gallian (2009). Two of the most important types of labeling are called graceful and harmonious, Graceful labeling were introduced independently by Rosa (1967) in 1966 and Golombo (1972) in 1972, while harmonious labeling were first studied by Graham and Sloane (1980). A third important type of labeling which contains aspects of both of the other two, is called cordial and was introduced by Cahit (1990). Whereas the label of an edge uv for graceful and harmonious labeling is given respectively by $|f(u) - f(v)|$ and $f(u) + f(v)(modulo q)$, cordial labeling use only labels 0 and 1 and the induced label $f(u) + f(v)(modulo 2)$, which is of course equals $|f(u) - f(v)|$. Because arithmetic modulo 2 is an integral part of computer science, cordial labeling have close connections with that field. More precisely, cordialgraphs are defined as follows.

***Corresponding author: Ponmoni, A.,**
Department of Mathematics, C.S.I. College of Engineering, Ketti – 643215, Tamilnadu, India

Definition 1.1

Let $G = (V, E)$ be an(p, q)- graph, let $f: V \to \{0,1\}$ and for each edge uv,assign the label $|f(u) - f(v)|$. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph is called cordial if it has a cordial labeling.

Definition 1.2

Let f be a function from the vertices of a graph $G to$ $\{0,1\}$and for each edge uv assign the label$|f(u) - f(v)|$. The function f is called a cordial labeling of G if $|v_f(0) - v_f(1)|$.

Definition 1.3

Let $G = (V, E)$ be an (p, q)- graph. A mapping$f: V \to \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f. For an edge$e = uv$, the induced edge labeling$f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$.Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively underf^*.

Graph labeling (Gallian, 2009) is a strong communication between number theory (David M. Burton, 1980) and structure of graphs (6). By combining the triangular number and divisibility concept in Number Theory and cordial labeling concept in graph labeling, we introduce a new concept called Triangular divisor cordial labeling. In this paper, we proved the standard graphs such as wheel(W_n), $K_{2,n} \odot u_2(K_1)$, and Subdivision of bistar $(< B_{n,n} : w >)$, Fan $(P_n \oplus K_1)$ are Triangular divisor cordial graphs. First we give the some concepts in Number Theory (Harary, 1972).

Definition 1.4

Let a and b be two integers. If a divides b means that there is a positive integer k such that b=ka. It is denoted by a/b. If a does not divide b, then we denote $a \nmid b$

Definition 1.5

The triangular number can be defined by

$$T_n = \binom{n+1}{2} n \geq 1$$

this generates the infinite sequence of integers beginning 1,3,6,10,15,21,28,36,45,55,66,78,91, ...

Main Results

Varatharajan, Navaneethakrishnan and Nagarajan (2011), introduced the notion of Divisor Cordial Labeling.

Definition 2.1

Let $G = (V, E)$ be a simple graph and $f : V \rightarrow \{1,2,3, \ldots, |V|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u) \not/ f(v)$. f is called a divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph (Varatharajan *et al*., 2011). Sridevi, Navaneethakrishnan introduced the notion of Fibonacci Divisor Cordial Labeling.

Definition 2.2

Let $G = (V, E)$ be a simple (p, q)- graph and $f : V \rightarrow \{F_1, F_2, F_3, \ldots, F_p\}$ where F_i is the i^{th} Fibonacci number, be a bijection. For each edge uv, assign the label 1 if either $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u) \nmid f(v)$. f is called a Fibonacci divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a Fibonacci divisor cordial labeling is called a Fibonacci divisor cordial graph [9].

These definitions motivate us to define a new type of cordial labeling called Triangular divisor cordial labeling as follows.

Definition 2.3

Let $G = (V, E)$ be a simple (p, q)- graph and $f : V \rightarrow \{T_1, T_2, T_3, \ldots, T_p\}$ where T_i is the i^{th} Triangular number, be a bijection. For each edge uv, assign the label 1 if either $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 otherwise. f is called a Triangular divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a Triangular divisor cordial labeling is called a Triangular divisor cordial graph.

Theorem: 2.4

Given a positive integer n, there is a Triangular divisor cordial graph G which has n vertices.

Proof

Define $f: V(G) \rightarrow \{T_1, T_2, \ldots, T_n\}$

Case: 1

n is even

Construct a path containing $\frac{n}{2} + 2$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}+2}$ which are labeled as $T_1, T_2, \ldots, T_{\frac{n}{2}+2}$ respectively. It is observed that $f(v_1 v_2) = 1 = f(v_2 v_3)$ and

$$f(v_i v_{i+1}) = 0 \quad where\ 3 \leq i \leq \frac{n}{2} + 1$$

Attach $\frac{n}{2} - 2$ vertics $v_{\frac{n}{2}+3}, v_{\frac{n}{2}+4}, \ldots, v_n$ with v_1 and they are labelled as $T_{\frac{n}{2}+3}, T_{\frac{n}{2}+4}, \ldots, T_n$ respectively. It is observed that $f(v_1 v_i) = 1 \quad where\ \frac{n}{2} + 3 \leq i \leq n$

Then we have $e_f(0) = \frac{n}{2} - 1$

$$e_f(1) = \frac{n}{2}$$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, the graph G is Triangular divisor cordial graph.

Case: 2

n is odd

Construct a path containing $\left\lceil\frac{n}{2}\right\rceil + 2$ vertices $v_1, v_2, \ldots, v_{\left\lceil\frac{n}{2}\right\rceil+2}$ which are labeled as $T_1, T_2, \ldots, T_{\left\lceil\frac{n}{2}\right\rceil+2}$ respectively. It is observed that $f(v_1 v_2) = 1 = f(v_2 v_3)$ and $f(v_i v_{i+1}) = 0 \quad where\ 3 \leq i \leq \left\lceil\frac{n}{2}\right\rceil + 1$

Attach $\frac{n}{2} - 3$ vertics $v_{\left\lceil\frac{n}{2}\right\rceil+3}, v_{\left\lceil\frac{n}{2}\right\rceil+4}, \ldots, v_n$ with v_1 and they are labelled as $T_{\left\lceil\frac{n}{2}\right\rceil+3}, T_{\left\lceil\frac{n}{2}\right\rceil+4}, \ldots, T_n$ respectively. It is observed that $f(v_1 v_i) = 1 \quad where\ \left\lceil\frac{n}{2}\right\rceil + 3 \leq i \leq n$

Then we have $e_f(0) = \left\lceil\frac{n}{2}\right\rceil - 1$

$$e_f(1) = \left\lceil\frac{n}{2}\right\rceil - 1$$

Therefore, $|e_f(0) - e_f(1)| \leq 1$
Hence, the graph G is Triangular divisor cordial graph.

Theorem: 2.5

The graph $C_4^{(t)}$ is Triangular divisor cordial graph.

Proof:

Let $V(C_4^{(t)}) = \{u_1^1 = u_1^2 = \ldots = u_1^t\} \cup \{u_j^i : 1 \leq i \leq t, 2 \leq j \leq 4\}$

And $E(C_4^{(t)}) = \{u_j^i u_{j+1}^i : 1 \leq j \leq 3\} \cup \{u_4^i u_1^i : 1 \leq i \leq t\}$

Then $|V(C_4^{(t)})| = 3t + 1$ *and* $|E(C_4^{(t)})| = 4t$

Define $f: V(G) \to \{T_1, T_2, \ldots, T_{3t+1}\}$ by

$f(u_1^i) = T_1, \quad 1 \leq i \leq t$
$f(u_j^i) = T_{2j-2}, \quad i = 1, 2 \leq i \leq 4$
$f(u_j^i) = T_{2j-1}, \quad i = 2, 2 \leq i \leq 4$
$f(u_j^i) = T_{3i+j-3}, \quad 3 \leq i \leq t, 2 \leq i \leq 4$

Then we have $e_f(0) = 2t$
$e_f(1) = 2t$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, The graph $C_4^{(t)}$ is Triangular divisor cordial graph.

Theorem: 2.6

The graph $G = K_{1,n}^+$ is triangular divisor cordial graph if $n \geq 2$.

Proof:

Let (V_1, V_2) be the bipartition of $K_{1,n}$ where $V_1 = \{u_0\}$ *and* $V_2 = \{u_1, u_2, \ldots, u_n\}$ and

$v_1, v_2, \ldots, v_n$ be the pendant vertices joined with $u_1, u_2, \ldots, u_n$ respectively and

$E(G) = \{u_0 u_i : 1 \leq i \leq n\} \cup \{u_i v_i : 1 \leq i \leq n\}$

Then $|V(G)| = 2n + 1$ *and* $|E(G)| = 2n$

Define $f: V(G) \to \{T_1, T_2, \ldots, T_{2n+1}\}$ by

$f(u_0) = T_1$
$f(u_i) = T_{i+1}, \quad i = 1,2$
$f(u_i) = T_{2i}, \quad 3 \leq i \leq n$
$f(v_1) = T_4$
$f(v_i) = T_{2i+1}, \quad 2 \leq i \leq n$

Then we have, $e_f(0) = n$
$e_f(1) = n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$
Hence, the graph $G = K_{1,n}^+$ is triangular divisor cordial graph if $n \geq 2$.

Theorem: 2.7

The graph $G = F_m \oplus K_{1,n}^+$ is triangular divisor cordial graph for all m and n.

Proof:

Let $V(G) = U \cup V$ where $U = \{u_0, u_1, u_2, \ldots, u_m\}$ be the vertices set of F_m and Let $V = (V_1, V_2)$

be the bipartition of $K_{1,n}^+$, where $V_1 = \{v = u_0\}$ *and* $V_2 = \{v_1, v_2, \ldots, v_n\}$ and Let $w_1, w_2, \ldots, w_n$ be the pendant vertices and joined these vertices with $v_1, v_2, \ldots, v_n$ respectively and
$E(G) = \{u_i u_{i+1} : 1 \leq i \leq m - 1\} \cup \{u_0 u_i : 1 \leq i \leq m\}$
$\cup \{v_0 v_i : 1 \leq i \leq n\} \cup \{v_i w_i : 1 \leq i \leq n\}$

So that $|V(G)| = 2m + n + 1$ *and* $|E(G)| = 2m + 2n - 1$

Define $f: V(G) \to \{T_1, T_2, \ldots, T_{m+2n+1}\}$ by

$f(u_0) = T_1$
$f(u_i) = T_{i+5}, \quad 1 \leq i \leq m$
$f(v_1) = T_2$
$f(v_2) = T_3$
$f(v_i) = T_{m+2i}, \quad 3 \leq i \leq n$
$f(w_1) = T_4$
$f(w_2) = T_5$
$f(w_i) = T_{m+2i+1}, \quad 3 \leq i \leq n$

Then we have, $e_f(0) = m + n - 1$
$e_f(1) = m + n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, the graph $G = F_m \oplus K_{1,n}^+$ is triangular divisor cordial graph for all m and n.

Theorem: 2.8

The graph $G = C_n @ K_{1,m}$ is triangular divisor cordial graph if $n > 4$ *and* $m = n - 3, n - 4, n - 5$.

Proof:

Let $V(G) = U \cup V$ where $U = \{u_1, u_2, \ldots, u_n\}$ be the vertices set of C_n and $V = \{v_1, v_2, \ldots, v_m\}$ be the vertex set of $K_{1,m}$.

$E(G) = \{u_1 v_i : 1 \leq i \leq m\}$

Define $f: V(G) \to \{T_1, T_2, \ldots, T_{m+n}\}$ by

$f(u_1) = T_1$
$f(u_i) = T_{i+1}, \quad i = 2,3,4$
$f(u_i) = T_{2i-4}, \quad 5 \leq i \leq n$
$f(v_1) = T_2$
$f(v_i) = T_{n+i}, \quad 2 \leq i \leq m$

Case: 1

$$m = n - 3$$

$e_f(1) = n - 1$
$e_f(0) = n - 2$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Case: 2

$$m = n - 4$$

$e_f(1) = n - 2$
$e_f(0) = n - 2$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Case: 3

$$m = n - 5$$

$e_f(1) = n - 3$
$e_f(0) = n - 2$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, The graph $G = C_n @ K_{1,m}$ is triangular divisor cordial graph if $n > 4$ and $m = n - 3, n - 4, n - 5$.

Theorem: 2.9

The graph $G = P_{m,n}$ is triangular divisor cordial graphif $m = 4$.

Proof:

Let $V(G) = \{u, v, u_i, v_i, w_i : 1 \leq i \leq n\}$
$E(G) = \{uu_i : 1 \leq i \leq n\} \cup \{u_i v_i : 1 \leq i \leq n\}$
$\cup \{v_i w_i : 1 \leq i \leq n\} \cup \{v w_i : 1 \leq i \leq n\}$

Define $f : V(G) \to \{T_1, T_2, \ldots, T_{3n+2}\}$ by

$f(u) = T_1$
$f(v) = T_2$
$f(u_i) = T_{3i}, \quad 1 \leq i \leq n$
$f(v_i) = T_{3i+1}, \quad 1 \leq i \leq n$
$f(w_i) = T_{3i+2}, \quad 1 \leq i \leq n$

Then we have, $e_f(0) = 2n$
$e_f(1) = 2n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, The graph $G = P_{4,n}$ is triangular divisor cordial graph.

Theorem: 2.10

The graph $G = B_{n,n}$ is triangular divisor cordial graph.

Proof:

Let u *and* v be the center vertices of $B_{n,n}$

Let $u_1, u_2, \ldots, u_n$ *and* $v_1, v_2, \ldots, v_n$ be the pendant vertices attached to the vertex u *and* v respectively.

Define $f : V(G) \to \{T_1, T_2, \ldots, T_{2n+2}\}$ by

$f(u) = T_1$ and $f(v)$ is not divisible by $f(v_1), f(v_2), \ldots, f(v_n)$
$f(u_1) = T_2$
$f(u_2) = T_3$
$f(u_i) = T_{2i+1}, \quad 3 \leq i \leq n$

and assign remaining triangular number to the vertices $f(v_1), f(v_2), \ldots, f(v_n)$

Then we have, $e_f(0) = n$
$e_f(1) = n + 1$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence, The graph $G = B_{n,n}$ is triangular divisor cordial graph.

REFERENCES

Cahit, I. 1990. On cordial and 3-equitable labelings of graphs, Utilitas Math, 370,189-198.

David M. Burton, 1980. Elementary Number Theory, Second Edition, Wm. C. Brown Company Publishers.

Gallian, J.A. 2009. A dynamic survey of graph labeling, Electronic Journal of combinatorics 16, DS6.

Golombo, S.W. 1972. How to number a graph in Graph Theory and Computing, R.C. Read, ed., Academic Press, New York, 23-27.

Graham, R.L. and N.J.A. Sloane, 1980. An additive bases and harmonious graphs, SIAM J. Alg. Discrete Math., 382-40

Harary, F. 1972. Graph theory Addition – Wesley, Reading, Mass.

Rosa, A. 1967. On certain valuations of the vertices of a graph, Theory of graphs (International Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Pairs, 39-355.

Varatharajan, R., S. Navaneethakrishnan and K. Nagarajan, 2011. Divisor cordial graph, *International Journal of Mathematical Comb*., Vol .4, 15-25.

Sridevi, R., S. Naveethakrishnan, 2013. Fibonacci divisor Cordialgraph, *International Journal of Mathematics and Soft Computing.*

Ponmoni, A., S. Navaneethakrishnan, A. Nagarajan submitted for publication.

****