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VARIATION ITERATION METHOD FOR SOLVING POROUS MEDIUM EQUATION

[1]*Alla M. Elsheikh and [2]Tarig M. Elzaki

[1,2]Department of Mathematics, Faculty of Science and Arts, Alkamil, University of Jeddah, Jeddah, Saudi Arabia
[1,2]Department of Mathematics, Faculty of Science, Sudan University of Science and Technology

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ABSTRACT

The aim of this paper, is to apply a new method called Variation Iteration perturbation method ((VIM)) to porous medium equation. This method is a combination of the new integral "Variation Iteration" and the perturbation method. The nonlinear term can be easily handled by perturbation method. The porous medium equations have importance in engineering and sciences and constitute a good model for many systems in various fields. Some cases of the porous medium equation are solved as examples to illustrate ability and reliability of mixture of Variation Iteration and perturbation method. The results reveal that the combination of Variation Iteration and perturbation method is quite capable, practically well appropriate for use in such problems and can be applied to other nonlinear problems. This method is seen as a better alternative method to some existing techniques for such realistic problems

INTRODUCTION

Many of the physical phenomena and processes in various fields of engineering and science are governed by partial differential equations. The nonlinear heat equation describing various physical phenomena called the porous medium equation. is where m is a rational number. There are number of physical applications where this simple model appears in a natural way, mainly to describe processes involving fluid flow, heat transfer or diffusion. In this paper, we apply a new method called Variation Iteration perturbation method ((VIM)) to solve porous medium equation. This method is a combination of the new integral "Variation Iteration" and the perturbation method.

The porous medium equation is:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{m}\frac{\partial u}{\partial x}\right) \tag{1}$$

Where m is a rational number.

The correction functional for the porous medium equation is

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t\lambda(\xi)\left[\frac{\partial u(x,\xi)}{\partial\xi}-u^{m}\frac{\partial^2\tilde{u}_n(x,\xi)}{\partial x^2}-mu^{m-1}\left(\frac{\partial u(x,\xi)}{\partial x}\right)^2\right]d\xi \tag{2}$$

***Corresponding author: Alla M. Elsheikh**
Department of Mathematics, Faculty of Science, Sudan University of Science and Technology

The Variation iteration method is used by applying two essential steps. It is required first to determine the Lagrange multiplier λ that can be identified optimally via integration by parts and $\tilde{u}_n$ is a restricted variation which means $\delta\tilde{u}_n = 0$. Having determined the Lagrange multiplier $\lambda(\xi)$, the successive approximations $u_{n+1}, n \geq 0$, of the solution u will be readily obtained upon using any selective function u_0. Consequently, the solution:

$$u = \lim_{n\to\infty} u_n$$

Applications

Now, we consider in this section the effectiveness of the Variation iteration method to obtain the exact or approximate analytical solution of the porous medium equations.

Example 3.1

Let us $m = -1$ in equation(1), we get,

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left(u^{-1}\right)\frac{\partial u}{\partial x}\right) \tag{3}$$

With the initial condition as $u(x,0) = \frac{1}{x}$

Solution

The correction functional for this equation is given by:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left[\frac{\partial u(x,\xi)}{\partial \xi} - u^{-1}\frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} + u^{-2}\left(\frac{\partial u(x,\xi)}{\partial x}\right)^2\right]d\xi$$

Where we used $\lambda = -1$ for first order porous medium equation as shown (3) we can use the initial condition to select $u_0(x,t) = u_0(x,0) = \frac{1}{x}$. Using this selection into the correction functional gives the following successive approximation

$$u_0(x,0) = \frac{1}{x}.$$

$$u_1(x,t) = u_0(x,t) - \int_0^t \lambda(\xi)\left[\frac{\partial u(x,\xi)}{\partial \xi} - u_0^{-1}\frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} + u_0^{-2}\left(\frac{\partial u(x,\xi)}{\partial x}\right)^2\right]d\xi = \frac{t}{x^2}$$

$$u_2(x,t) = u_1(x,t) - \int_0^t \lambda(\xi)\left[\frac{\partial u_1(x,\xi)}{\partial \xi} - u_1^{-1}\frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} + u_1^{-2}\left(\frac{\partial u(x,\xi)}{\partial x}\right)^2\right]d\xi = \frac{t^2}{x^3}$$

Proceeding in similar manner we can obtain further values, we get solution in the form of a series

$$u(x,t) = \frac{1}{x} + \frac{t}{x^2} + \frac{t^2}{x^3} + \frac{t^3}{x^4} \dots \dots = \frac{1}{x-t} \tag{4}$$

This is the solution of (3) and which is exactly the exact solution give above

Example 3.2

Let us $m = 1$ in equation (1), we get:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) \qquad ,u(x,0)=x \tag{5}$$

Solution

The correction functional is:

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t \lambda(\xi)\left[\frac{\partial u(x,\xi)}{\partial \xi}-u\frac{\partial^2\tilde{u}_n(x,\xi)}{\partial x^2}-\left(\frac{\partial u_n(x,\xi)}{\partial x}\right)^2\right]d\xi$$

Consider $\lambda=-1$, and $u_0(x,t)=u_0(x,0)=x$

Then: $u_1(x,t)=u_0(x,t)-\int_0^t\left[\frac{\partial u_0(x,\xi)}{\partial \xi}-u_0\frac{\partial^2\tilde{u}_0(x,\xi)}{\partial x^2}-\left(\frac{\partial u_0(x,\xi)}{\partial x}\right)^2\right]d\xi=x+t$

$$u_2(x,t)=u_1(x,t)-\int_0^t\left[\frac{\partial u_1(x,\xi)}{\partial \xi}-u_1\frac{\partial^2\tilde{u}_1(x,\xi)}{\partial x^2}-\left(\frac{\partial u_1(x,\xi)}{\partial x}\right)^2\right]d\xi$$

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$$u(x,t)=x+t+0+0=x+t$$

Example 3.3

Let us $m=-4/3$ in equation (1), we get:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{-4/3}\frac{\partial u}{\partial x}\right) \tag{6}$$

With initial condition as $u(x,0)=(2x)^{-3/4}$

Solution

The correction functional is:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t\left[\frac{\partial u(x,\xi)}{\partial \xi}-u^{-4/3}\frac{\partial^2\tilde{u}_n(x,\xi)}{\partial x^2}+\frac{4}{3}u^{-7/3}\left(\frac{\partial u_n(x,\xi)}{\partial x}\right)^2\right]d\xi$$

Consider $\lambda=-1$, and $u(x,0)=(2x)^{-3/4}$

$$u_1(x,t)=u_0(x,t)-\int_0^t\left[\frac{\partial u_0(x,\xi)}{\partial \xi}-u_0^{-4/3}\frac{\partial^2\tilde{u}_0(x,\xi)}{\partial x^2}+\frac{4}{3}u_0^{-7/3}\left(\frac{\partial u_0(x,\xi)}{\partial x}\right)^2\right]d\xi$$

$$=(2x)^{-3/4}-\int_0^t\left[\tfrac{-21}{4}(2x)^{-\frac{7}{4}}+3(2x)^{-\frac{7}{4}}\right]d\zeta=(2x)^{-3/4}+9\times2^{-15/4}\times x^{-7/4}\times t$$

$$u_2(x,t)=u_1(x,t)-\int_0^t\left[\frac{\partial u_1(x,\xi)}{\partial \xi}-u_1^{-4/3}\frac{\partial^2\tilde{u}_1(x,\xi)}{\partial x^2}+\frac{4}{3}u_1^{-7/3}\left(\frac{\partial u_1(x,\xi)}{\partial x}\right)^2\right]d\xi=189\times2^{-31/4}x^{-11/4}\times t^2$$

Then : $u(x,t)=(2x)^{-3/4}+9\times2^{-15/4}\times x^{-7/4}\times t+189\times2^{-31/4}x^{-11/4}\times t^2$

This result can be verified through substitution.

Conclusion

In this paper we show that the applicability of the mixture of "Variation Iteration" with the homotopy perturbation method to construct an analytical solution for porous medium equation. This combination of two methods successfully worked to give very reliable and exact solutions to the equation. This method provides an analytical approximation in a rapidly convergent sequence with in exclusive manner computed terms. Its rapid convergence shows that the method is trustworthy and introduces a significant improvement in solving nonlinear partial differential equations over existing methods.

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