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MULTI-CRITERIA GROUP DECISION MAKING FOR PIPE MATERIAL SELECTION: COMPARATIVE ANALYSIS OF HF-VIKOR AND HF-ELECTRE II

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ABSTRACT

The material has an important role in an engineering design field. The suitable material selection for a particular product is one of the multiple criteria decision making (MCDM) problem depends on an aggregation representing by closeness to the ideal which is generated by compromise methods. In order to complete the product requirements, experts need to analyze the performance of various and suitable materials with precise operations. In the competitive market, the material selection process is intricate and time consuming effort. There is a need to choose an efficient approach for the selection of best alternative material of a product. The purpose of this study is to solve a MCDM problem of pipe material selection in a group, in addition, give a comparative analysis of hesitant fuzzy VIKOR (HF-VIKOR) and the hesitant fuzzy ELECTREE II (HF-ELECTREE II). An example of pipe material selection of Jamal Din Wali sugar industry in Pakistan is conducted to illustrate the application of the proposed approach; finally, the ranking performance of these MCDM methods is also compared with each other.

INTRODUCTION

In the development of any structural element, material selection is one of the most challenging issues and it is also critical for the success and to meet the demands of cost reduction and better performance. In our study the material selection is one of the most challenging MCDM issues and it is also critical for the success and to meet the demands of cost reduction and better performance. Generally, experts are choosing a material by adopting the trial and error methods with investment of huge cost or build on collection of past data leading to less of time (Shanian and Savadogo, 2006). While selecting alternative materials, a clear understanding of functional needs for each individual component is required and various important criteria need to be considered. An improper selection can negatively affect productivity, profitability and reputation of an organization (Karande and Chakraborty, 2012). The complex inter-relationships between variety of materials and its selection criteria frequently make the material selection process a difficult and time consuming task.

In the literature many researchers have been studied for example, Sapuan *et al*. (2002) presented a prototype knowledge based system (KBS) for material selection in the engine components while, Sapuan (2001) proposed KBS in the domain of polymeric based composite material selection process. Findik and Turan (2012) presented the weighted property index (WPIM) method to select the best material for lighter wagon design and the results shows aluminum alloy is the opt material for lighter wagons. Ramalhete *et al*. (2010) used the digital tool for material selection problem. Enab and Bondok (2013) integrated the finite element method for choosing the suitable material for designing the tibia component of cemented artificial. Ipek *et al*. (2013) presented the materials selection problem in the manufacturing field using expert system model. Fayazbakhsh *et al*. (2009) attempted the Z-transformation method for selecting the suitable material for cryogenic storage tank and so on.

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One of the most important stages in material selection process is ranking and selecting the suitable material for a particular application. MCDM methodologies are rapidly growing in the material selection problem. Because these techniques or methods has been used to provide a better aspect to solve practical problems in daily life. In general, the decision makers used different layout and descriptions to express their preferences or choices for each alternative in a group decision making problem (Xu, 2008; Massanet *et al*., 2014). In the literature a large number of studied has been conducted in MCDM for material selection problems. For example, Holloway (1998) integrated the importance of material selection in engineering applications and also enlightened the impact of environment due to improper selection of material. Shanian and Savadogo (2006) proposed the Elimination and Choice Expressing the Reality (ELECTRE) model for selecting suitable material for loaded thermal conductorChatterjee *et al*. (2009) integrated a compromised ranking and outranking method for material selection problem. Bahraminasab and Jahan (2011) integrated a comprehensive VIKOR method to material selection for femoral component of knee replacement in medical field.

Jahan *et al*. (2011) used a VIKOR method for selecting the suitable material for rigid pin of shaft. Hambali *et al*. (2009) discussed theimportance of Analytical Hierarchy Process (AHP) in material selection problem. Maitya and Chakraborty (2013) proposed Fuzzy TOPSIS method to select the suitable abrasive material for grinding wheel. Jahan and Edwards (2013) used VIKOR method for material selection problem with interval numbers and target based criteria and so on. From above literature survey, a systematic and efficient approach for material selection is necessary in order to select the best alternative for a product. Thus the great efforts need to be extended to determine criteria that influence material selection for a product to eliminate unsuitable alternatives and select the apt material alternative using simple and logical methods. The appropriate material for different application is identified and selected using different MCDM methods. The application of MCDM for proposed the suitable material for sugar industry equipment is also one among of them. The previous studies in sugar industry have been proposed and used the various anti corrosive medium and coating material on the critical equipment of sugar industry.

But problem of failures are not omitted completely. In sugar industry many pipe lines are damaged due to acidic nature of sugarcane juice. Keeping in view of the evidence the decision making drives to increase the difficulty in selection of the appropriate material. However, there is few papers discussed MCGDM case for suitable pipe material selection in sugar industries. This study focused on the development and application of MCGDM techniques under hesitant fuzzy environment for selection of suitable material for pipes. In addition, give a comparative analysis of extended VIKOR and ELECTRE II method under hesitant fuzzy environment, called HF-VIKOR and HF-ELECTRE II, in which the difference of opinions among group members is taken into account by HFSs. In rest of this paper is organized as follows, in section 2 some preliminaries are described. In section 3, the frame work of pipe material selection is given. Proposed approach is demonstrated by using an illustrative problem in section 4. Finally conclusions are given in section 5.The aim of this paper is the selection of best pipe material in Jamal Din Wali (JDW) sugar industry which is situated in Pakistan.

Preliminaries

According to Torra and Narukawa (2009) and Torra (2010)all previous extensions of fuzzy sets are based on same rationalities that's all are not clearly assign the membership degree of an element to a fixed set, so they proposed a new generalized form of fuzzy set called hesitant fuzzy set (HFS). After that it was attracted more and more researchers (Xia and Xu (2011a); Rodriguez *et al*. (2012); Xu and Xia, M. (2011b); Liao *et al*. (2014).Torra (2010) firstly gave the concept of HFS, defined some of its basic operations. Torra and Narukawa (2009) also presented an extension principle permitting to generalize the existing operations on fuzzy sets to HFS, and described the application of this new type of sets in the framework of group decision making. Xia and Xu (2011) gave the complete idea of original definition of HFS with including mathematical representation, stated as follows:

Definition 1 Let X be a fixed set, a hesitant fuzzy set on X in terms of a function that when applied to X returns a subset of [0,1]. For better understanding a mathematical form can be presented in the following terms:

$$E = \{\langle x, h_E(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}, \qquad (1)$$

Where $h_E(x)$ denotes the set of some different values between 0 and 1and it is also denoting the possible membership degrees of an element $x \in X$ to the set E.

For three hesitant fuzzy sets, Torra and Narukawa (2009) and Torra, V. (2010), after thatXia and Xu(2011)presents some operations laws that are given as follows:

Let h, h_1 and h_2 be a three hesitant fuzzy elements then their some operations are defined as follows, here λ is a positive real number.

(1) Lower bound: $h^-(x) = \min h(x)$ or $\min \{ \gamma \mid \gamma \in h \}$;
(2) Upper bound: $h^+(x) = \max h(x)$ or $\max \{ \gamma \mid \gamma \in h \}$;
(3) Intersection: $\tilde{h}_1 \cap \tilde{h}_2 = \cup_{\gamma_1 \varepsilon \tilde{h}_1, \gamma_2 \varepsilon \tilde{h}_2} \min\{\gamma_1, \gamma_2\}$;

(4) Union: $\tilde{h}_1 \cup \tilde{h}_2 = \cup_{\gamma_1 \varepsilon \tilde{h}_1, \gamma_2 \varepsilon \tilde{h}_2} \max\{\gamma_1, \gamma_2\}$;

(5) $h^{\lambda} = \cup_{\gamma_1 \varepsilon h_1} \{\gamma^{\lambda}\}$;

(6) $\lambda h = \cup_{\gamma_1 \varepsilon h_1} \{1-(1-\gamma)^{\lambda}\}$;

(7) $h_1 \oplus h_2 = \cup_{\gamma_1 \varepsilon h_1, \gamma_2 \varepsilon h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;

(8) $h_1 \otimes h_2 = \cup_{\gamma_1 \varepsilon h_1, \gamma_2 \varepsilon h_2} \{\gamma_1\gamma_2\}$;

Some different types of distance measures for HFS are also presented in literature, but in our study we use some of them, stated as follows:

Definition 2 (Xu and Xia 2011b) Let h_A and h_B be two hesitant fuzzy sets on the X = x_1, x_2, $x_3 \ldots x_n$, then the hesitant Euclidean distance of HFs

$$d(h_A, h_B) = \sqrt{\frac{1}{L}\sum_{j=1}^{L} | h_{A\rho(j)} - h_{B\rho(j)} |^2} \quad (2)$$

Here L is the number of elements in the HFEs h_A, and h_B.

Definition 3 (Xu and Xia (2011c) Let h_A and h_B be two hesitant fuzzy sets on the values $X = x_1, x_2, x_3, \ldots, x_n$, then hesitant Manhattan distance of two HFEs are represented as follows:

$$d(h_A - h_B) = \frac{1}{L}\sum_{j=1}^{L} | h^j{}_{A\rho} - h^j{}_{B\rho} | \quad (3)$$

Here L is the number of values in HFEs of h while $h^j{}_{A\rho}$ and $h^j{}_{B\rho}$ are the j_{th} largest values in h_A and h_B.

In these definitions, in many situations or cases $L(h_A) \neq L(h_B)$ for better understanding we can write, $L = \max L(h_A), L(h_B)$. For obtaining the correct results we will extend the shorter or greater element until all of them have same length. By adding any value we can extend shorter set, but the best way is to add the same value several times. The selection of these values depends on decision makers if the decision makers are pessimistic then they will increase less value while if the decision makers are optimistic then they will increase greater value.

Example: Let h_A = (0.3,0.4,0.5) and h_B = (0.6,0.7) be two HFEs, the length of both sets are not equal so we can extend h_B set by increasing the shortest value and the new h_B = (0.6, 0.6, 0.7). The Manhattan distance of h_A and h_B are calculated as follows:

$$d(h_A, h_B) = \frac{|0.3-0.6| + |0.4-0.6| + |0.5-0.7|}{3} = 0.233$$

Definition 4 (Xia Xu 2011a) For hesitant fuzzy element (HFE)

$$S_{fun}(h) = \frac{1}{L_h}\sum_{\gamma \varepsilon h} \gamma \quad (4)$$

called the score function of HFE h. in this equation the L_h is a total number of h values. For two hesitant fuzzy elements $h_A$and h_B, if $S_{fun}(h_A) > S_{fun}(h_B)$, then $h_A > h_B$; if $S_{fun}(h_A) = S_{fun}(h_B)$, then $h_A = h_B$.

In some special cases, for the purpose of comparison it is difficult or it cannot be distinguishing the two HFEs. Liao *et al*. (2014) introduce the variance function of HFEs to overcome this issue and then offered a novel method to rank these elements.

Definition 5 For a hesitant fuzzy element (HFEs)

$$v(h_\delta)=\frac{1}{L_h}\sqrt{\sum_{\gamma_i,\gamma_j \varepsilon h}(\gamma_i-\gamma_j)^2} \quad (5)$$

the variance function of h, where L_h is the number of value in $h$and $V(h_\delta)$ is called the variance degree of h. For two HFEs h_A and h_B, if $V(h_A) > V(h_B)$, then $h_A < h_B$; if $V(h_A) = V(h_B)$, then $h_A = h_B$.

The relationships between both functions are similar to the relationship between mean and variance in statistics. Any two HFEs can be comparing easily from the following functions which are based on the score function and the variance functions.

If $S(h_{fun(A)}) < S(h_{fun(B)})$, then $h_A < h_B, MAX\{h_A, h_B\} = h_B$ and $MIN\{h_A, h_B\} = h_A$;
If $v(h_{\delta(A)}) = v(h_{\delta(B)})$, then $h_A = h_B$, and $MAX\{h_A, h_B\} = MIN\{h_A, h_B\} = h_A = h_B$

Definition 6 (Chen and Xu 2015) **For** a HFE h, the deviation degree $\sigma(h)$ of h can be expressed by

$$\delta(\mathrm{h}) = \sqrt{\frac{1}{L_h}\sum_{\gamma \in h}(\gamma - S_{fun}(\mathrm{h}))^2} \quad (6)$$

Here, $\delta(\mathrm{h})$ denote the conventional standard variance in statistics. It reflects the deviation degree between all values in a HFEs and their average value. A small $\delta(\mathrm{h})$ shows that the numerical values in h approach each other, meaning a high consistency of opinions among different DMs.

Frame work for pipe material selection

The suitable pipe material selection in sugar industries are very difficult MCGDM problem. In our defining problem, let E_g, $(g = 1,2,3, \cdots, G)$ be the committee of various experts. E that contains a discrete set of m alternatives, expressed as $A_{t_1} = \{A_1, A_2, \ldots, A_m\}$. Let $C_j = \{c_1, c_2, \ldots, c_n\}$ be the set of all criteria. A HFS A_{t_1} is defined on C is given by $A_{t_1} = \{< c_j, \mathrm{h}^\rho_{A_{t_1}}(\mathrm{c}_j) > | \mathrm{c}_j \in C\}$, where $h_{mt_1}(\mathrm{c}_j) = \{\gamma \mid \gamma \in h^\rho_{A_{t_1}}(\mathrm{c}_j), 0 \le \gamma \le 1\}$; $t_1 = 1, 2 \ldots m$ and $j = 1, 2, \ldots, n$. Here $h^\rho_{A_{t_1}}(\mathrm{c}_j)$ represents the possible membership degree of the t^{th} alternative A_{t_1} satisfying the j^{th} criterion c_j and can be expressed as a HFE $h_{t_1 j}$. The hesitant fuzzy decision matrix is defined as:

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \quad (7)$$

Given that each criterion has different importance, the weight vector of all criteria is defined as $W = (w_1, w_2, \ldots, w_n)^T$, where $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$ with w_j denoting the importance degree of the criterion c_j. The complete computation procedure to solve a MCGDM problem is given in Fig. 1

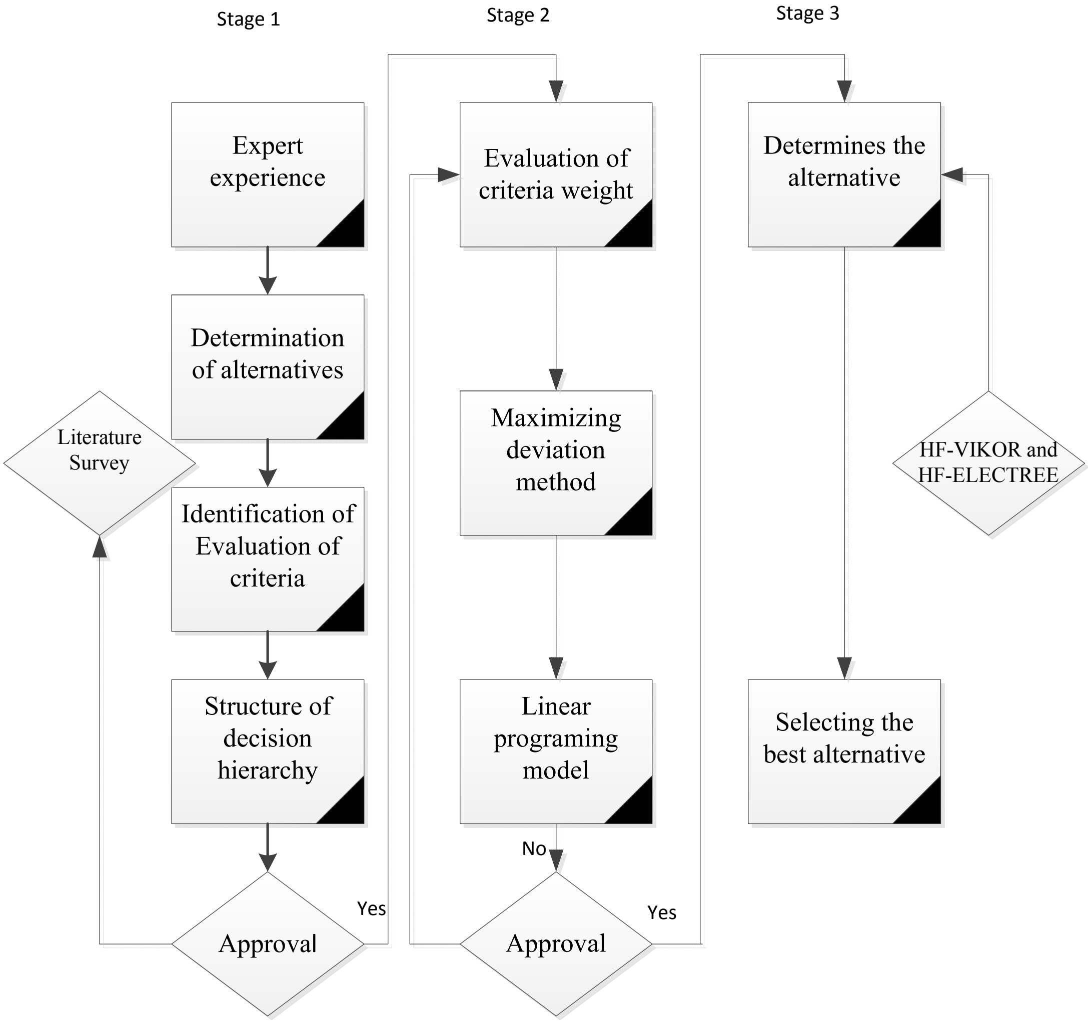


Fig. 1. Schematic figure of the propose model for pipe material selection

Computation of criteria weight

The evaluation of the criteria weights plays an important role in MCDM or MADM. Yingming (1997) proposed the maximizing deviation method to evaluate the criteria weights for solving the MCDM problems with numerical information. In a MCDM problem, the criteria with a larger deviation value between the alternatives should be assigned a larger weight, while the criteria with a small deviation value between the alternatives should be signed a smaller weigh (Yingming 1997). So in the process of ranking the alternatives, if one criteria has similar criteria values across the alternatives, it should be gave a small weight; Otherwise, the criteria which makes larger deviations should be estimated a larger weight, in spite of the degree of its own importance. Especially, if all available alternatives score are equal with respect to a given criteria, then such a criteria will be judged not important by most of the decision makers According to Yingming (1997).the zero weight should be assigned to the corresponding criterion. However, there is a situation that the information about the criteria weight is partially known. For this case, Xu and Zhang (2013) constructed liner programming model based on the maximizing deviation method to evaluate the optimal relative weights of criteria under hesitant fuzzy information. The constrained optimization model is given as follows:

$$\begin{cases} \max \quad d(\mathrm{w}) = \sum_{j=1}^{n}\sum_{t_1=1}^{m}\sum_{t_2=1}^{m} w_j \sqrt{\frac{1}{L}\sum_{l=1}^{L} | h_{t_1 j}^{\rho} - h_{t_2 j}^{\rho} |^2} \\ s.t \quad w \in \Delta,\ w_j \geq 0,\ j = 1,2,...,\mathrm{n},\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{8}$$

Here Δ is a set of constraint condition that the weight value satisfy the according to the situation. The information about eight vectors is partially known and the known information is given as follows:

$$\Delta = \{0.05 \le w_1 \le 0.11, 0.12 \le w_2 \le 0.16, 0.10 \le w_3 \le 0.17, 0.13 \le w_4 \le 0.18, 0.1 \le w_5 \le 0.19,$$
$$0.11 \le w_6 \le 0.17, 0.10 \le w_7 \le 0.30, \quad \sum_{j=1}^{7} w_j = 1$$

The modal 3.2 is a linear programming modal that can be solved LINGO software. We can get the optimal values of $w = (\mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_n)^T$ from the solving of this modal; these weighted values can be used as the weight vector of the criteria.

Aggregation methods

In our study we propose the comparative study of H-F VIKOR and H-F ELECTREE method to solve the MCGDM problem. We also give the basic idea of these two methods in our procedure:

HF-VIKOR method for MCGDM

In decision making process, sometime it is very difficult or impossible for decision makers or experts to determine the exact values of the criteria because of uncertainty and hesitancy. In .this situation the hesitant fuzzy sets are very powerful tool to deal the uncertainty and hesitancy. So there is more appropriate to consider the values of the criteria as hesitant fuzzy element, where the hesitant fuzzy elements are the benefit criteria. The H-F VIKOR methods for MCGDM method have been present, from these steps some are presented by Zhang and Wei (2013).

The HF-VIKOR method can be described as Algorithm 1 which has the following steps:

Step 1: Arranging the committee of decision making group and defining a finite set of criteria and alternatives.

Step 2: In this step the decision maker's aggregate weights of j_{th} criteria. The fuzzy weight of each criterion is calculated from the maximizing deviation method mentioned as above.

Step3: determine the positive ideal solution (PIS) and the negative ideal solution for all criteria, when j is associated with benefit criteria, it follows that:

$$A_j^* = \{h_1^*, h_2^*, ..., h_n^*\} \tag{9}$$

Where

$$h_j^* = \cup_{t_1=1}^{m} h_{t_1 j}^{\rho} = \bigcup_{\gamma_{1j} \varepsilon h_{1j}, ..., \gamma_{mj} \varepsilon h_{mj}} \max\{\gamma_{1j}, \gamma_{2j}, ..., \gamma_{mj}\}, j=1,2,...,n$$

$$A_j^- = \{h_1^-, h_2^-, ..., h_n^-\} \tag{10}$$

Where

$$h_j^- = \cup_{t_1=1}^{m} h_{t_1 j}^{\rho} = \bigcup_{\gamma_{1j} \in h_{1j}, ..., \gamma_{mj} \varepsilon h_{mj}} \min\{\gamma_{1j}, \gamma_{2j}, ..., \gamma_{mj}\}, j=1,2,...,n$$

Step4: Compute the normalized hesitant fuzzy difference $d_{t_1 j}, t_1 = 1,2,...,\mathrm{m}, j=1,2,...,\mathrm{n}$. The benefit criteria $d_{t_1 j}$ is calculated from Eq. 2.10, as follows:

$$d_{t_1 j} = \frac{\sqrt{\frac{1}{L}\sum_{j=1}^{L} | h_j^* - h_{ij} |^2}}{| h_j^* - h_j^- |} \tag{11}$$

Step5: Compute the index S_{t_1} and R_{t_1} over the benefit criteria as follows:

$$S_{t_1} = \sum_{j=1}^{n} \{w_j * d_{t_1 j}\} \tag{12}$$

$$R_{t_1} = \max_j \{w_j * d_{t_1 j}\} \quad (13)$$

The index S_{t_1} represents the hesitant fuzzy group utility measure and the index R_{t_1} represents the hesitant fuzzy individual regret measures.

$$Q_{t_1} = v\frac{S_{t_1} - S_{\min}}{S_{\max} - S_{\min}} + (1-v)\frac{R_{t_1} - R_{\min}}{R_{\max} - R_{\min}} \quad (14)$$

Where

$$S_{\max} = \max_{t_1} S_{t_1}, S_{\min} = \min_{t_1} S_{t_1},$$

$$R_{\max} = \max_{t_1} R_{t_1}, R_{\min} = \min_{t_1} R_{t_1},$$

and v is the strategy weight of maximum group utility while 1-v shows the weight of individual regret.

Step 7: Rank the alternatives sorting by the values *S*, *R* and *Q* in ascending order. The results are three ranking lists $\{A\}_S$, $\{R\}_R$, $\{A\}_Q$.

Step 8: Propose a compromise solution. The alternative denoted as $A^{(1)}$ which is the best ranked by the measure *Q* (minimum) is considered as a compromise solution if the following two conditions are satisfied:

Cond1: If: $Adv = \geq DQ$

$Adv = Q(A^{(1)})-Q(A^{(2)}) \geq 1/(m-1)$

Where *Adv* is the advantage of the alternative $A^{(1)}$ ranked first, $A^{(2)}$ is the alternative with the second position in $\{A\}_Q$ and $DQ = 1/(m-1)$ is the threshold.

Cond2: Acceptable Stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by *S* or/and *R*.
If one of the two conditions is not satisfied, then a set of compromise solution is proposed.

The HF-ELECTRE II methods for MCGDM

To solve the MCGDM problem, we develop the idea of HFSs with ELECTRE II method. It formulates the traditional method in to a new approach, called hesitant fuzzy ELECTRE II (HF-ELECTRE II. The procedure of HF-ELECTREE has following steps, some of them proposed by Chen and Xu (2015). This method also can be described as Algorithm 2.

Step1: In this step arranging the committee of decision makers to determine the relevant criteria of the potential alternatives and also give the evaluation information in the form of hesitant fuzzy element set of the alternative with respect to the given criteria. They also calculate the importance weight vector $w = (w_1, w_2, w_3, ..., w_n)^T$ for the relevant criteria calculated from the maximizing deviation method (mentioned as above), and the relative weight vector $\omega = (\omega_C, \omega_{C'}, \omega_{C''}, \omega_D, \omega_{D'}, \omega_{D''}, \omega_{X^=})^T$

Step2: In this step calculate the score function $S_{fun}(h)$ and deviation function $\delta(h)$ of each evaluation information of *h* value of each alternatives under the criteria are as follows from the definition 4 and 6 respectively. In this step, the $S_{fun}(h)$ and $\delta(h)$ are worked together to compare different alternatives on a criterion, if any alternative has higher $S_{fun}(h)$ or lower $\delta(h)$ it means that alternative is better than others. In the case where the alternatives have the same score the higher the value of $S_{fun}(h)$, the larger the membership degree while the small $\delta(h)$ means a lower hesitancy degree between decision makers.

Step3: Calculate the hesitant fuzzy concordance sets (it can be classified as the hesitant fuzzy strong, medium and weak concordance sets) on the basis of score function and deviation function. For each pair of the alternatives A_{t_1} and ($t_1, t_2 = 1, 2... m$), the hesitant fuzzy concordance sets of these twos pairs are the sum of those criteria where the performance of A_{t_1} is superior to A_{t_2}. There are three types of classification are given as follows:

- The hesitant fuzzy strong concordance set $J_{C_{t1t2}}$

$$J_{C_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) > S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) < \delta(h^{\rho}_{t_2 j})$$

- The hesitant fuzzy medium concordance set $J_{c'_{kl}}$

$$J_{C'_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) > S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) \geq \delta(h^{\rho}_{t_2 j})$$

- The hesitant fuzzy weak concordance set $J_{c''_{kl}}$

$$J_{C''_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) = S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) < \delta(h^{\rho}_{t_2 j})$$

Where *J = { j | j = 1, 2, . . ., n}* represents a set of subscripts of all criteria.

The three types of hesitant fuzzy concordance sets exhibit the different degrees that A_{t_1} is superior to A_{t_2}. It is the deviation function that reflects the main difference between $J_{c_{t1t2}}$ and $J_{c'_{t1t2}}$. Moreover, a lower deviation value shows that the opinions of the DMs have a higher consistency degree. Thus, $J_{c_{t1t2}}$ is more concordant than $J_{c'_{t1t2}}$. Relative to the deviation function, the score function plays an important role in determining the magnitude of HFEs. Hence $J_{c'_{t1t2}}$ having a higher score value is more concordant than $J_{c''_{t1t2}}$

Step4: Construct the hesitant fuzzy discordance sets and it also based (it can be classified as the hesitant fuzzy strong, medium and weak discordance sets) on the score function and deviation function. For the hesitant fuzzy discordance sets the all criteria for which A_{t_1} is inferior to A_{t_2}. They are also defined in to three steps:

- The hesitant fuzzy strong discordance set $J_{D'_{t1t2}}$

$$J_{D_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) < S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) > \delta(h^{\rho}_{t_2}) \}$$

- The hesitant fuzzy medium discordance set $J_{D''_{kl}}$

$$J_{D'_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) < S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) \leq \delta(h^{\rho}_{t_2 j}) \}$$

- The hesitant fuzzy weak discordance set $J_{D'''_{kl}}$

$$J_{D''_{t1t2}} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) = S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) > \delta(h^{\rho}_{t_2 j}) \}$$

Step5: If $S_{fun}(h^{\rho}_{t_1 j}) = S(h^{\rho}_{t_2 j}) \text{and} \sigma(h^{\rho}_{t_1 j}) = \sigma(h^{\rho}_{t_2 j})$, in the hesitant fuzzy concordance and discordance sets then we define this case as follows:

- A hesitant fuzzy indifferent set and express it by

$$j^{=}_{t_1 t_2} = \{ j \mid S_{fun}(h^{\rho}_{t_1 j}) = S_{fun}(h^{\rho}_{t_2 j}) \text{and} \delta(h^{\rho}_{t_1 j}) = \delta(h^{\rho}_{t_2 j}) \}$$

Step 6: Construct the hesitant fuzzy concordance index and also obtain the concordance matrix. The hesitant fuzzy concordance indices are defined as the ratio of the sum of the weights related to the criteria in the hesitant fuzzy concordance and indifferent sets and to that of all criteria. The concordance index $c_{t_1t_2}$ of A_{t_1} and A_{t_1} in the HF-ELECTRE II method are computed as:

$$C_{t1t2}=\frac{\omega_c \times \sum_{j\in J_{c_{t1t2}}} w_j + \omega_{c'} \times \sum_{j\in J_{c'_{t1t2}}} w_j + \omega_{c''} \times \sum_{j\in J_{c''_{t1t2}}} w_j + \omega_{j^=} \times \sum_{j\in J^=_{t1t2}} w_j}{\sum_{j=1}^{n} w_j}$$

$$=\omega_c \times \sum_{j\in J_{c_{t1t2}}} w_j + \omega_{c'} \times \sum_{j\in J_{c'_{t1t2}}} w_j + \omega_{c''} \times \sum_{j\in J_{c''_{t1t2}}} w_j + \omega_{j^=} \times \sum_{j\in J^=_{t1t2}} w_j \quad (15)$$

Here w_j denotes the weight of the criterion c_j, satisfying $\sum_{j=1}^{n} w_j = 1$ for the normalized weight vector of all criteria. $\omega_c, \omega_{c'}, \omega_{c''}$ and $\omega_{j^=}$ are the attitude weights of hesitant fuzzy strong, medium and weak concordance sets and the weights of the hesitant fuzzy indifferent sets depending on the attitudes of the DMs. The $c_{t_1t_2}$ show the relative importance of A_{t_1} with respect to A_{t_1} is $0 \le c_{t_1t_2} \le 1$. A large $c_{t_1t_2}$ value means that the alternative A_{t_1} is superior to the alternative A_{t_1}. The hesitant fuzzy concordance matrix C can thus be constructed by using the obtained value of the indices $c_{t_1t_2}$ $(\mathrm{t}_1, \mathrm{t}_2 = 1,2,...,m; \mathrm{t}_1 \ne t_2)$ as:

$$C=\begin{bmatrix} - & \cdots & c_{1t_2} & \cdots & c_{1(\mathrm{m}-1)} & c_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ c_{t_1 1} & \cdots & c_{t_1t_2} & \cdots & c_{\mathrm{t}_1(\mathrm{m}-1)} & c_{t_2 m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ c_{m1} & \cdots & c_{mt_2} & \cdots & c_{m(\mathrm{m}-1)} & - \end{bmatrix} \quad (16)$$

Step 7: Calculate the hesitant fuzzy discordance index based on the weighted distance and also obtain the discordance matrix. The hesitant fuzzy discordance index reflects the relative difference of A_{t_1} with respect to A_{t_2} in terms of discordance criteria and is defined by the following equation:

$$d_{t_1t_2}=\frac{\max_{j\in J_{D_{t1t2}} \cup J_{D'_{t1t2}} \cup J_{D''_{t1t2}}} \{\omega_D \times d(\mathrm{w}_j \mathrm{h}^{\rho}_{t_1j}, \mathrm{w}_j \mathrm{h}^{\rho}_{t_2j}), \omega_{D'} \times d(\mathrm{w}_j \mathrm{h}^{\rho}_{t_1j}, \mathrm{w}_j \mathrm{h}^{\rho}_{t_2j}), \omega_{D''} \times d(\mathrm{w}_j \mathrm{h}^{\rho}_{t_1j}, \mathrm{w}_j \mathrm{h}^{\rho}_{t_2j})\}}{\max_{j\in J} d(\mathrm{w}_j \mathrm{h}^{\rho}_{t_1j}, \mathrm{w}_j \mathrm{h}^{\rho}_{t_2j})} \quad (17)$$

Here $\omega_D, \omega_{D'}$ and $\omega_{D''}$ are respectively the weights of three types of hesitant fuzzy discordance sets, which depend on the DMs attitudes. $d(\mathrm{w}_j \mathrm{h}^{\rho}_{t_1j}, \mathrm{w}_j \mathrm{h}^{\rho}_{t_2j})$ is distance measure.

Similarly, with the hesitant fuzzy discordance indices for all pair wise comparisons of alternatives, the hesitant fuzzy discordance matrix can be formulated by

$$D=\begin{bmatrix} - & \cdots & d_{1t_2} & \cdots & d_{1(\mathrm{m}-1)} & d_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ d_{t_1 1} & \cdots & d_{t_1t_2} & \cdots & d_{\mathrm{t}_1(\mathrm{m}-1)} & d_{t_1 m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ d_{m1} & \cdots & d_{mt_1} & \cdots & d_{m(\mathrm{m}-1)} & - \end{bmatrix} \quad (18)$$

The elements of D are different from those of C. The former stands for the relative difference of $w_j h^{\rho}_{t_1j}$ for all hesitant fuzzy discordance indices; that is, it reflects the limited compensation between alternatives, namely, when the difference of two alternatives on a criterion reaches a certain extent, the compensation of the loss on a given criterion by a gain on another one may not be acceptable for DMs (Bouyssou 1986; Figueira *et al*. (2005).

Step 8: Construct the outranking relations from the given concordance and discordance levels for each pair of alternatives. There are two types of outranking relations; a strong relationship S^R and a weak relationship S^r, which are constructed by comparing of concordance and discordance levels. A strong relationship leads to a good discrimination among the alternatives and thus yields a more refined and stricter ranking procedure than the weak relationship Duckstein and Gershon 1983; Hokkanen *et al*. (1995). To define the two relationships, let c^*, c^0, c^-be three decreasing levels of concordance index, which are denoted by $0 < c^- < c^0 < c^* < 1$. Also, let d^0 and d^*represent two increasing levels of discordance satisfying $0 < d^0 < d^* < 1$. With these specifications, $A_{t_1} S^R A_{t_2}$ is defined if and only if one or both of the following sets of conditions holds

$$(1)\begin{cases} C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{c}^* \\ D(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \leq \mathrm{d}^* \\ C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{C}(\mathrm{A}_{t_2},\mathrm{A}_{t_1}) \end{cases}$$

$$(2)\begin{cases} C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{c}^0 \\ D(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \leq \mathrm{d}^0 \\ C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{C}(\mathrm{A}_{t_2},\mathrm{A}_{t_1}) \end{cases} \quad (19)$$

The weak relationship $A_{t_1} S^r A_{t_2}$ is defined if and only if the following conditions hold:

$$\begin{cases} C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{c}^- \\ D(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \leq \mathrm{d}^* \\ C(\mathrm{A}_{t_1},\mathrm{A}_{t_2}) \geq \mathrm{C}(\mathrm{A}_{t_2},\mathrm{A}_{t_1}) \end{cases} \quad (20)$$

Step 9: Draw the strong and weak outranking graphs and obtain the final ranking of all alternatives. As a result of the two pair wise outranking relationships, the strong graphs and the weak graphs are respectively constructed for the strong relationship and for the weak relationship. These graphs will be used in an iterative procedure to obtain the desired ranking of the alternatives. Specifically, the ranking procedure consists of a forward ranking v' a reverse ranking $v''$and an average ranking $v(\mathrm{x})\left[=\frac{v'+v''}{2}(\mathrm{x})\right]$. Finally, we rank the alternatives according to the values of *v(x)*.

A numerical application of proposed model

The proposed models have been applied to solve a practical problem in the sugar industry located at District Rahim Yar Khan, Province Punjab, Pakistan. The average sugarcane crushing capacity of 126 days is 37652 tons per day. The cane sugar production processes involves various stages like reception, cleaning, extraction, juice clarification, evaporation, crystallization, centrifugation, drying, storing and packing. The piping has a major role to bridge the various stages of the production process. The acid nature of the sugar cane juice is corroding the inner surface of the pipe. It leads to the frequent maintenance of the pipe lines and which may interrupt the production. The industrial persons are taking an effort to overcome the aforementioned problem to replace the existing material with suitable one. The management makes a committee of five experts or engineers E_g (g=1,2..,G); they proposed five alternate A_{t1} (t_1=1,2,..,m) and seven influencing criteria $C_j(j=1,2,..n)$. The influencing criteria described as (C_1) = Hardness of pipe (HP), (C_2) = Pipe tensile strength (PTS), (C_3) = Pipe yield strength (PYS), (C_4) = Pipe revisibility (PR), (C_5) = Manufacturability (M), (C_6) = Permissible pressure (PP), (C_7) = Thermal conductivity (TC), which are extracted from literature (Goel *et al*., 2007; Wesley *et al*., 2012; Prado *et al*., 2010) and experts survey To avoid psychic contagion, the decision makers are required to provide their preferences in anonymity. Fig. 2 show the decision hierarchy of proposed selection and the proposed approaches are utilized to solve this MCGDM with the following steps:

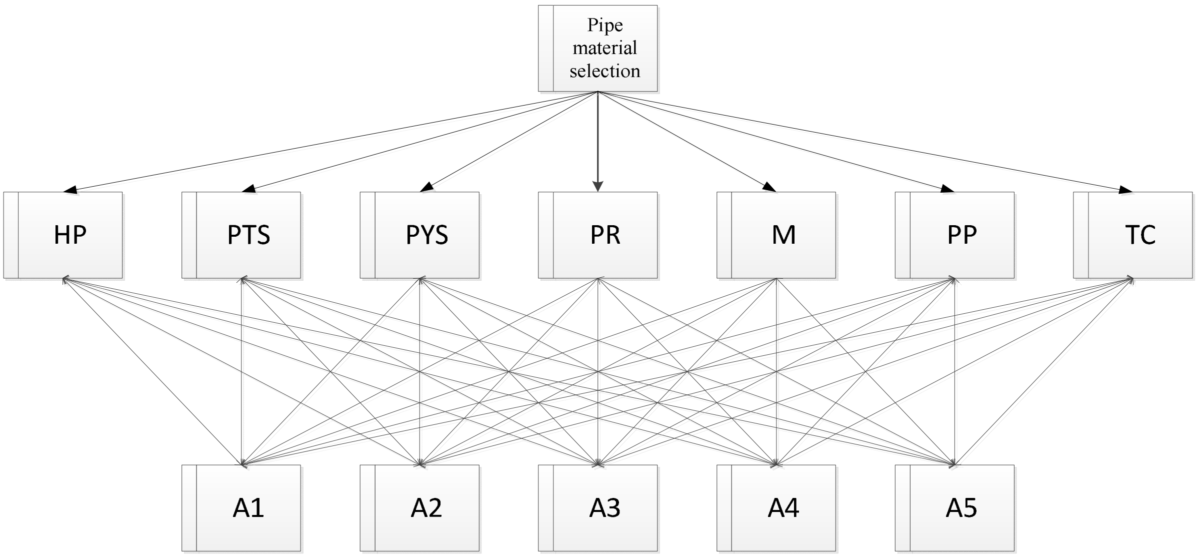


Fig. 2. Decision hierarchy of pipe material selection

The solution approach of Algorithm1 is stated as follows:

Step 1: Making a committee of decision makers and describing a finite set of criteria and alternatives then give an aggregated hesitant fuzzy decision matrix.

Table 1. Aggregated hesitant fuzzy decision matric

	A_1	A_2	A_3	A_4	A_5
C_1	{0.5,0.6,0.7}	{0.2,0.3,0.6,0.7}	{0.3,0.4,0.6}	{0.2,0.3,0.6}	{0.4,0.5,0.6,0.7}
C_2	{0.4,0.6,0.7,0.9}	{0.5,0.6,0.8}	{0.7,0.8}	{0.3,0.4,0.6,0.7}	{0.7,0.9}
C_3	{0.2,0.4,0.7}	{0.2,0.6,0.8}	{0.2,0.3,0.6,0.7,0.9}	{0.3,0.4,0.5,0.7,0.8}	{0.2,0.4}
C_4	{0.3,0.6,0.8,0.9}	{0.2,0.4,0.6}	{0.3,0.6,0.7,0.9}	{0.2,0.4,0.5,0.6}	{0.5,0.7,0.8,0.9}
C_5	{0.4,0.5,0.6,0.8}	{0.3,0.6,0.7,0.8}	{0.2,0.3,0.5,0.6}	{0.1,0.3,0.4}	{0.3,0.5,0.6}
C_6	{0.2,0.5,0.6,0.7}	{0.3,0.6,0.8}	{0.3,0.4,0.7}	{0.2,0.3,0.7}	{0.4,0.5}
C_7	{0.2,0.3,0.4,0.6}	{0.1,0.4}	{0.4,0.5,0.7}	{0.2,0.5,0.6}	{0.2,0.3,0.7}

Step 2: In this step the decision maker's aggregate weights of j_{th} criteria. The weigh vector of each criterion by utilized the linear programing Eq. (8) given as follows:

$$\begin{cases} Max \quad D(\mathrm{w}) = 4.3267*\mathrm{w}_1 + 8.9946*\mathrm{w}_2 + 3.8224*w_3 + 3.6004*w_4 + 3.8224*w_5 + 3.9946*w_6 + 1.6004*w_7 \\ S.t \qquad w \in \Delta, \qquad w_j \geq 0, \qquad j=1,2,3,4,5,6,7. \end{cases}$$

By solving this linear model, we get optimal weight vector $w = (0.11, 0.16, 0.17, 0.13, 0.16, 0.17, 0.10)^T$.

Step 3: From Table 3.1, calculate the best value and the worst value of each criteria according to Eq. (9) and (10) respectively. The results are shown as follows:

$$\{h_1^* = 0.7, h_2^* = 0.9, h_3^* = 0.9, h_4^* = 0.9, h_5^* = 0.8, h_6^* = 0.7, h_7^* = 0.7\}$$
$$\{h_1^- = 0.2, h_2^- = 0.3, h_3^- = 0.2, h_4^- = 0.2, h_5^- = 0.1, h_6^- = 0.2, h_7^- = 0.1\}$$

Step 4: Compute the normalized hesitant fuzzy difference, $d_{t_1 j}$, $t_1 = 1,2, \cdots ,n$, $j = 1,2, \cdots ,m$. For benefit criteria d_{11} is calculated from Eq. (11) as follows:

$$d_{21} = \frac{\sqrt{\frac{1}{5}\{|0.7-0.2|^2 + |0.7-0.3|^2 + |0.7-0.6|^2 + |0.7-0.7|^2 + |0.7-0.7|^2\}}}{|0.7-0.2|} = 0.5797$$

Step 5: The values S_{tl} *and* R_{tl} are calculated respectively according to Eqs. (12) and (13).

$$S_1 = w_1 * d_{11} + w_2 * d_{12} + w_3 * d_{13} + w_4 * d_{14} + w_5 * d_{15} + w_6 * d_{16} + w_7 * d_{17} = 0.4096$$
$$S_2 = 0.4636, S_3 = 0.4539, S_4 = 0.5825, S_5 = 0.3577$$
$$R_1 = \max_4 \{w_1 * d_{11} + w_2 * d_{12} + w_3 * d_{13} + w_4 * d_{14} + w_5 * d_{15} + w_6 * d_{16} + w_7 * d_{17}\} = 0.0959$$
$$R_2 = 0.0938, R_3 = 0.1015, R_4 = 0.1026, R_5 = 0.0934$$

Step 6: Compute the values Q_{tl} for each alternative with Eq. (14):

$$Q_1 = 0.2513, Q_2 = 0.2572, Q_3 = 0.6542, Q_4 = 1, Q_5 = 0$$

Table 2. Ranking order sorting by *S, R* and *Q* values

	A_1	A_2	A_3	A_4	A_5	The ranking order
S	0.4096	0.4636	0.4539	0.5825	0.3577	$A_5 \succ A_1 \succ A_3 \succ A_2 \succ A_4$
R	0.0959	0.0938	0.1015	0.1026	0.0934	$A_5 \succ A_2 \succ A_1 \succ A_3 \succ A_4$
Q	0.2513	0.2572	0.6542	1	0	$A_5 \succ A_1 \succ A_2 \succ A_3 \succ A_4$

Step 7: According to Table 2, rank the alternatives sorting by the values *S, R* and *Q* in ascending order and the results are shown in Table 2.

Step 8: Verify if the two conditions are satisfied or not. Since

$Q(A^{(2)}) - Q(A^{(1)}) = 0.0059 \leq 1 / (5\text{-}1) = 0.25$

the Cond1 is not satisfied. Further, we have $Q(A^{(2)}) - Q(A^{(1)}) = DQ = 0.25$ which suggest that the compromise solution should include all five alternatives. This analysis shows that the H-F VIKOR method is simpler and recognize the alternative A_5 is a best ranked by S or/and R. In order to make a comparative analysis, the HF-ELECTRE II is utilized to tackle the same problem and steps of Algorithm2 can be defined as follows:

Step1: In this step arranging the committee of decision makers to determine the relevant criteria of the potential alternatives and give an aggregated hesitant fuzzy decision matrix. They calculate the weight vector for the relevant criteria same like step 2 of Algorithm 1. The DMs also assign the relative attitude weights of strong, medium and weak hesitant fuzzy concordance, discordance and indifferent sets as: $\omega = (\omega_c, \omega_{c'}, \omega_{c''}, \omega_D, \omega_{D'} \omega_{D''}, \omega_{J^=}) = (1, 0.9, 0.8, 1, 0.7, 0.9, 0.8)$

Step 2: Calculate the score value and the deviation value of each alternative with respect to each criterion with Eq. (4) and Eq. (6); see Table 3 and 4.

Table 3. Score values calculated from the score function

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.6	0.65	0.4333	0.65	0.575	0.50	0.375
A_2	0.45	0.6333	0.5333	0.40	0.60	0.5667	0.25
A_3	0.4333	0.75	0.54	0.625	0.40	0.4667	0.5333
A_4	0.3667	0.5	0.54	0.425	0.2667	0.40	0.4333
A_5	0.55	0.80	0.30	0.725	0.4667	0.45	0.40

Table 4. Deviation values calculated from the deviation function

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.0816	0.1803	0.2054	0.2791	0.1478	0.1871	0.1469
A_2	0.2062	0.1248	0.2493	0.1633	0.1871	02054	0.1500
A_3	0.1248	0.0500	0.2577	0.2164	0.1581	0.3350	0.1248
A_4	0.1699	0.1581	0.1855	0.1478	0.1248	0.2160	0.1700
A_5	0.1118	0.1000	0.1000	0.1478	0.1248	0.0500	0.2160

Step3: Construct the hesitant fuzzy strong, medium, and weak concordance set, indicating by J_c, $J_{c'}$ and $J_{c''}$ respectively, according to step 3 of Algorithm 2:

$$J_{c_{kl}} = \begin{bmatrix} - & 1,7 & 1,5,6 & 1,6 & 1 \\ - & - & 6 & 2,6 & - \\ 2,7 & 2,7 & - & 1,2,7 & 7 \\ 3 & 3,4 & - & - & 7 \\ 2,4 & 1,2,4 & 1,4,5 & 1,2,6 & - \end{bmatrix}, \quad J_{c'_{kl}} = \begin{bmatrix} - & 2,4 & 4 & 2,4,5 & 3,5,6 \\ 3,5,6 & - & 1,5 & 1,5 & 3,5,6 \\ 3 & 3,4 & - & 4,5,6 & 3,6 \\ 7 & 7 & - & - & 3 \\ 7 & 7 & 2 & 4,5 & - \end{bmatrix}$$

$$J_{c''_{kl}} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & 3 & - & - \\ - & - & - & - & - \end{bmatrix}$$

Step4: Construct the hesitant fuzzy strong, medium and weak discordance sets, indicating by J_D, $J_{D'}$ and $J_{D''}$ respectively according to step 4 of Algorithm 2:

$$J_{D_{kl}} = \begin{bmatrix} - & - & 2,7 & 3 & 2,4 \\ 1,7 & - & 2,7 & 3,4 & 1,2,4 \\ 1,5,6 & 6 & - & - & 1,4,5 \\ 1,6 & 2,6 & 1,2,7 & - & 1,2,6 \\ 1 & - & 7 & 7 & - \end{bmatrix}, \quad J_{D'_{kl}} = \begin{bmatrix} - & 3,5,6 & 3 & 7 & 7 \\ 2,4 & - & 3,4 & 7 & 7 \\ 4 & 1,5 & - & - & 2 \\ 2,4,5 & 1,5 & 4,5,6 & - & 4,5 \\ 3,5,6 & 3,5,6 & 3,6 & 3 & - \end{bmatrix}$$

$$J_{D''_{kl}} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & 3 & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

Step 5: Construct the hesitant fuzzy indifferent set, indicating by $J^{=}$, according to step 5 of Algorithm 2:

$$J^{=} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

Step 6: Calculate the hesitant fuzzy concordance index with Eq. (15), and also construct the concordance matrix according to step 6 of Algorithm 2:

$$C=(c_{t_1,t_2})_{5\times5}=\begin{bmatrix} - & 0.4710 & 0.5570 & 0.6850 & 0.5600 \\ 0.4500 & - & 0.4130 & 0.5730 & 0.4500 \\ 0.4300 & 0.5300 & - & 0.7840 & 0.4060 \\ 0.2700 & 0.3900 & 0.3060 & - & 0.2530 \\ 0.3800 & 0.4900 & 0.5440 & 0.7010 & - \end{bmatrix}$$

Step 7: Construct the hesitant fuzzy discordance index with Eq. (16), and the construct the discordance matrix according to step 7 of Algorithm 2:

$$D=(d_{t_1t_2})_{5\times5}=\begin{bmatrix} - & 0.5735 & 1 & 0.3091 & 1 \\ 0.7423 & - & 0.7281 & 0.4958 & 0.4958 \\ 1 & 0.7000 & - & 0.2782 & 0.4278 \\ 0.7000 & 0.7000 & 1 & - & 1 \\ 0.4659 & 0.4165 & 0.3987 & 0.3825 & - \end{bmatrix}$$

Step 8: Calculate the outranking relations with the given concordance and discordance levels. Which are chosen by decision makers as: $(c^-, c^0, c^*) = (0.55, 0.65, 0.70)$ and $d^0 = 0.45$, $d^* = 0.50$. According to Eqs (19) and (20), the outranking relations are derived and also as shown in Table 5.

Table 5. Out ranking relations

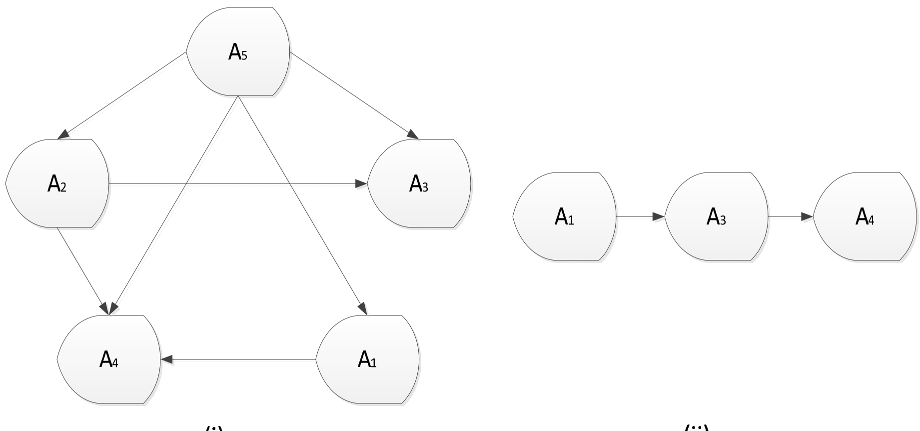
	A_1	A_2	A_3	A_4	A_5
A_1		-	S^F	S^F	
A_2				-	
A_3	-		S^f	S^F	
A_4			-	S^f	
A_5	S^F	S^F	S^F	S^F	-

Step 9: Plot the weak and strong outranking graphs show Fig. 3 and follow the exploration procedure according to step 9 of Algorithim 2, indicated above, the forward ranking v′, the reverse ranking v′′ and the average ranking ῡ are deduced and summarized in Table 6. From Table 6, the final ranking of the five alternatives is:

$$A_5 \succ A_1 \succ A_3 \succ A_2 \succ A_4$$

Table 3. 6 Result of ranking

	A_1	A_2	A_3	A_4	A_5
Forward ranking v′	2	3	4	5	1
Reverse ranking v′′	3	2	4	5	1
Average ranking ῡ	2.5	2.5	4	5	1



(i) (ii)

Fig. 3. Outrinking graph (i) Strong outranking (ii) weak outranking

Step 5: Conclusion and discussion

Ranking results of both Algorithms are similar, since they are based on the similar decision foundation by considering both maximum group utility and minimum individual regret. The compromise solution by HF-ELECTRE II provide a balance between a maximum group utility of the majority, obtained by concordance while a minimum of individual regret of the opponent obtained by discordance. But the HF-ELECTRE II is complex and time consuming method as compare to HF-VIKOR. No doubt, sugar industry has a vital role towards an economic development of the nation. The sugar industry is a challenging and repairable engineering industry which comprises of various systems including feeding, juice extraction, steam generation, refining, and crystallization.

The efficient operation of the industry needs to reduce and provide prolonged life of the pipes. The proper material selection plays a predominant role in the failures of the pipes in sugar industry. This study has presented a MCGDM methods based on combining of HF-VIKOR and HF-ELECTRE II to evaluate suitable material for pipes. The linear programing model is applied to compute the weights of evaluation criteria. A case example is illustrated for examining the results of the proposed model. This study involves various evaluation criteria so the MCDM techniques are producing significant results and also a bridge the gap in between the past research in sugar industry for material selection problem.

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