



Full Length Research Article

THE REVIEWS OF EINSTEIN'S EQUATION OF LOGARITHMIC DISTRIBUTION PLATFORM AND THE PROCESS OF CHANGES IN THE SPEED RANGE OF THE KARKHEH RIVER, KHUZESTAN PROVINCE, IRAN

¹Kaveh Ostad-Ali-Askari and ²Mohammad Shayannejad

¹Department of Water Engineering, Faculty of Civil Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Isfahan, Iran

²Department of Water Engineering, Isfahan University of Technology, Isfahan, Iran

ARTICLE INFO

Article History:

Received 16th December, 2014
Received in revised form
24th January, 2015
Accepted 27th February, 2015
Published online 31st March, 2015

Key words:

Velocity profile,
Gradation curve,
Einstein velocity distribution equation
Karkheh River

ABSTRACT

In this study in order to evaluate the accuracy of the equation and its coefficient variation in addition to obtain flow characteristics in six sections and estimates of geometric parameters using the SPSS software HEC-RAS 4, sampled in many points of bed soil and were used in analysis. Results showed that the bed was rough and this roughness was only a function of the high slope of waterway and velocity gradient predicted by equation has good agreement with obtained gradient. High slope of this gradient near bed represents instability of bed and sediment analysis performed by the software HEC-RAS 4 also confirmed upstream erosion.

Copyright © 2015 Kaveh Ostad Ali Askari and Mohammad Shayannejad. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

INTRODUCTION

Precise study of hydrodynamics, erosion and sedimentation and diffusion of materials requires us to use the physical models or two-dimensional or three-dimensional mathematical models in. In addition to the basic equations related to flow that their solution are possible only with the digital computer techniques and manly by imprecise numerical methods. Experimental relations can be used for initial estimation of flow behavior and flow characteristics such as depth, speed and discharge of a waterway (Aksoy and Levent Kavvas, 2005). In the present study, we assessed one of the beneficial vertical velocity distributions as called the logarithmic relationship of Einstein velocity distribution. The reason for choosing this relationship for study is its appropriate accuracy in prediction of stream velocity-depth profiles on smooth surfaces, interstitial and rough hydrodynamic surfaces.

$$\frac{u}{v} = 5.75 \log \frac{12.27 y}{\Delta} \quad (1)$$

$$\Delta = \frac{D_{65}}{x} \quad (2)$$

Where u is the point velocity, u^* shear rate and y is vertical distance of investigated point to bottom of the waterway. X coefficient is obtained through graph and as a function of $\frac{D_{65}}{\delta^2}$, δ is thickness of the lower laminar layer Figure 1.

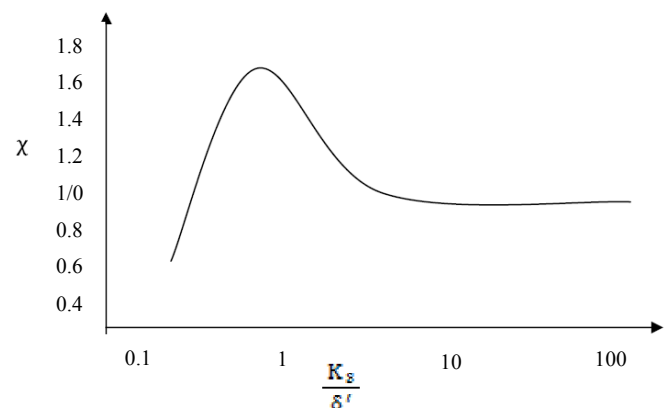


Figure 1. Einstein Diagram

*Corresponding author: Kaveh Ostad-Ali-Askari
Department of Water Engineering, Faculty of Civil Engineering,
Najafabad Branch, Islamic Azad University, Najafabad, Isfahan, Iran

$$\delta^- = \frac{11.6 v}{U_*} \tag{3}$$

X factor depending on the roughness of the bottom varies between 0.69 to 1.63 and for the hydrodynamic rough surfaces is equal to 1. Thus can be said Δ value is equal to D_{65} for the rough surfaces. The aim of this study was evaluation of the equation accuracy and estimation of Δ coefficient in velocity distribution equation and whether the correction factor Δ (X) change in the range of studies result or not. This velocity distributions equation can be used easily by finding Δ changes in period of study (Einstein and Harder, 1954).

MATERIALS AND METHODS

The theoretical equations of velocity distribution

In general, we faced to flow in waterways that its boundary layer is turbulent. Turbulent boundary layer is more complex than laminar boundary layer. In laminar lower layer, the velocity distribution is based on Newton's law of viscosity that is related to shear stress and viscosity. Assuming that the shear is almost constant and equal to stress on wall (τ), we have:

$$\tau_0 = \mu \frac{du}{dy} \tag{4}$$

Where μ is viscosity, $\left(\frac{du}{dy}\right)$ is velocity gradient and u is velocity at the point with y distance from the wall (boundary). After integrating the last equation under boundary condition ($u=0, y=0$) the following result is obtained:

$$u = \frac{\tau_0 y}{\mu} \tag{5}$$

In attention to $(V_0 = \sqrt{\frac{\tau_0}{\rho}})$ and $\left(\frac{\mu}{\rho}\right)$ we have:

$$\frac{u}{V_*} = \frac{V_* y}{\nu} \tag{6}$$

Out of the laminar lower layer, flow is turbulent, so there is a different form of the velocity distribution. In this layer, in addition to the viscosity force of the layer due to the mixing of the fluid particles, small whirlpools and cross currents, other shear stress that is called apparent shear stresses or Reynolds stresses arise. Thus, according to the theory of Prandtl mixing length and Carmen similarity theory for turbulence can be written as:

$$\tau = \rho K^2 Y^2 \left(\frac{du}{dy}\right)^2 \tag{7}$$

Where: τ is shear stress at the point where its distance from the y plate and velocity gradient is $\left(\frac{du}{dy}\right)$ at that point. Based on measurements, shear stress in the proximity of plate can be equal to shear stress on the plate ($\tau = \tau_0$), thus:

$$\tau_0 = \rho K^2 Y^2 \left(\frac{du}{dy}\right)^2 \tag{8}$$

And because $(V_*^2 = \frac{\tau_0}{\rho})$, and approximately $k = 0.4$, we have:

$$\frac{du}{dy} = 2.5 V_* \frac{V_*}{y} \tag{9}$$

After integration of the recent equation under the condition $u = 0$ at $y = y'$, the result would be:

$$\frac{u}{V_*} = 2.5 \ln \frac{y}{y'} = 5.75 \log \frac{y}{y'} \tag{10}$$

In the last equation y' is the integration constant and u is velocity at the point where its distance from solid boundary is y . Constant value of y' must be determined by empirical data, that its value is different depending on the boundary is smooth or rough. The last equation is known as wall logarithmic law and when credit that $\frac{y}{\delta} < 0.2$. In $\frac{y}{\delta} < 0.2$, firstly the shear stress is nearly constant and equal to the boundary shear stress (τ_0), secondly, the effect of pressure gradient is negligible. This region ($0 < \frac{y}{\delta} < 0.2$) is often called internal region. Out of this region ($0.2 \leq \frac{y}{\delta} \leq 0.8$) is called external region. Internal region includes a laminar lower layer and velocity distribution is logarithmic in this region. The thickness of the inner region in each place is approximately 20% of the thickness of boundary layer (Wu and Moin, 2009 and Bhattacharyya *et al.*, 2011). Nikuradze for smooth and rough surfaces (pipe and channel) after determining the integral constant (y') in the above equation based on the experimental results presented the following equation for the velocity distribution in the internal area:

$$\frac{u}{V_*} = 5.75 \log \frac{V_* y}{\nu} + 5.5 \text{ for smooth border} \tag{11}$$

$$\frac{u}{V_*} = 5.75 \log \frac{V_* y}{K_S} + 8.5 \text{ for rough border} \tag{12}$$

The velocity distribution equation is similar to above equation for the boundary that is interstitial the smooth and rough (interstitial boundary), except that constant value of relationship changes between 8.6 to 5.8. The results observed in the outer region of the boundary layer ($0.2 \leq \frac{y}{\delta} \leq 1$) in waterways show that we can use reduction velocity law for the velocity distribution in this region. According to the reduction velocity law, velocity distribution equation for smooth and rough surfaces is:

$$\frac{U_{max} - u}{V_*} = 5.75 \log \frac{\delta}{y} + A \tag{13}$$

Where U_{max} is velocity at the outer edge of the boundary layer and δ is boundary layer thickness and A is numeric value that must be determined by experiment. Qligan obtained following relationships for u velocity:

$$\frac{u}{V_*} = 5.75 \log \frac{V_* y}{\nu} + A_S \text{ for smooth channel} \tag{14}$$

$$\frac{u}{V_*} = 5.75 \log \frac{y}{K_S} + A_R \text{ for rough channel} \tag{15}$$

In recent equations, A_S and A_R are constant values that their values were determined 3.25 and 6.25, respectively, by Qligan.

Experimental results showed that A_S and A_R values are not constant value and depend on Froude number. The results of this analysis demonstrate for Froude numbers less than 4, A_S value remains constant, so the above equation can be used in most practical program, with assuming that A_S and A_R are constant. Above equations cannot be used to waterways that their bottom is interstitial hydro-dynamically. Einstein and Barbarossa based on Nikuradze study, presented the following equation to determine the speed of the channels with smooth, Interstitial and coarse surface.

$$\frac{u}{v_*} = 5.75 \log \frac{12.27 y}{\Delta} \quad (16)$$

$$\Delta = \frac{D_{65}}{X} \quad (17)$$

In the above equation, X is correction factor that is the function of D_{65} / δ and its value must be determined from the curve. Therefore, the above equation along with the curves can be used to any surface (Denicol *et al.*, 2010 and Huai *et al.*, 2009).

Studies range from the foot bridge to Shush, Khuzestan province, Iran

Study area was selected ranges of Karkheh River with a length of 41 km from the foot bridge to Shush, Khuzestan province, Iran. 6 sections in interval, 2 sections at the end of interval and 4 sections in the intermediate interval were selected. Sections were recorded by echo sounder and total station camera RTS538 over 3 weeks. Taken sections were introduced to HEC-RAS4 software to obtain fastly geometric parameters in each section in different surface water balance Interval scheme shown in Figure 2.

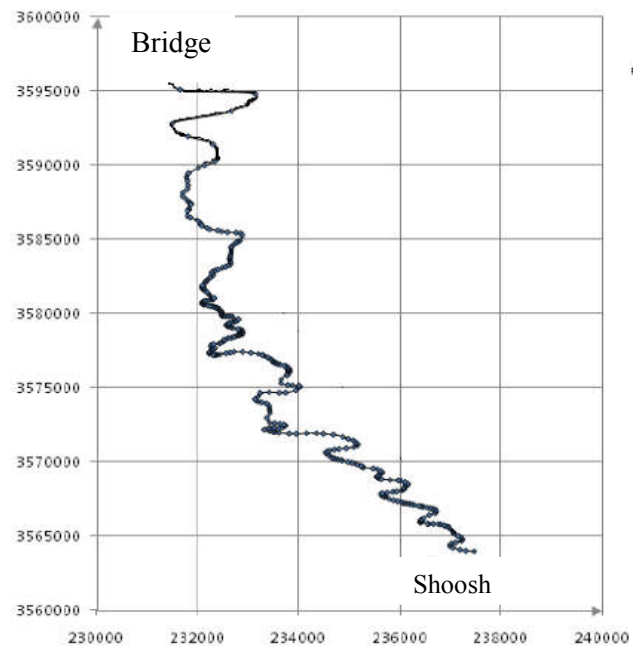


Figure 2. In the fields of bridge-foot interval of Shush, Khuzestan province, Iran

After marking sections, flow characteristics were obtained in studies section during a few weeks. Obtained characteristics including the measurement of the mean velocity by dividing each section to 5 subsection and estimating the average speed of each subsection by three-point method using the Molinet, measurement the slope of the water surface, estimate the geometric parameters such as cross section, wetted area and the water level width for each section by software geometric characteristics of taken sections are displayed in Figure 3.

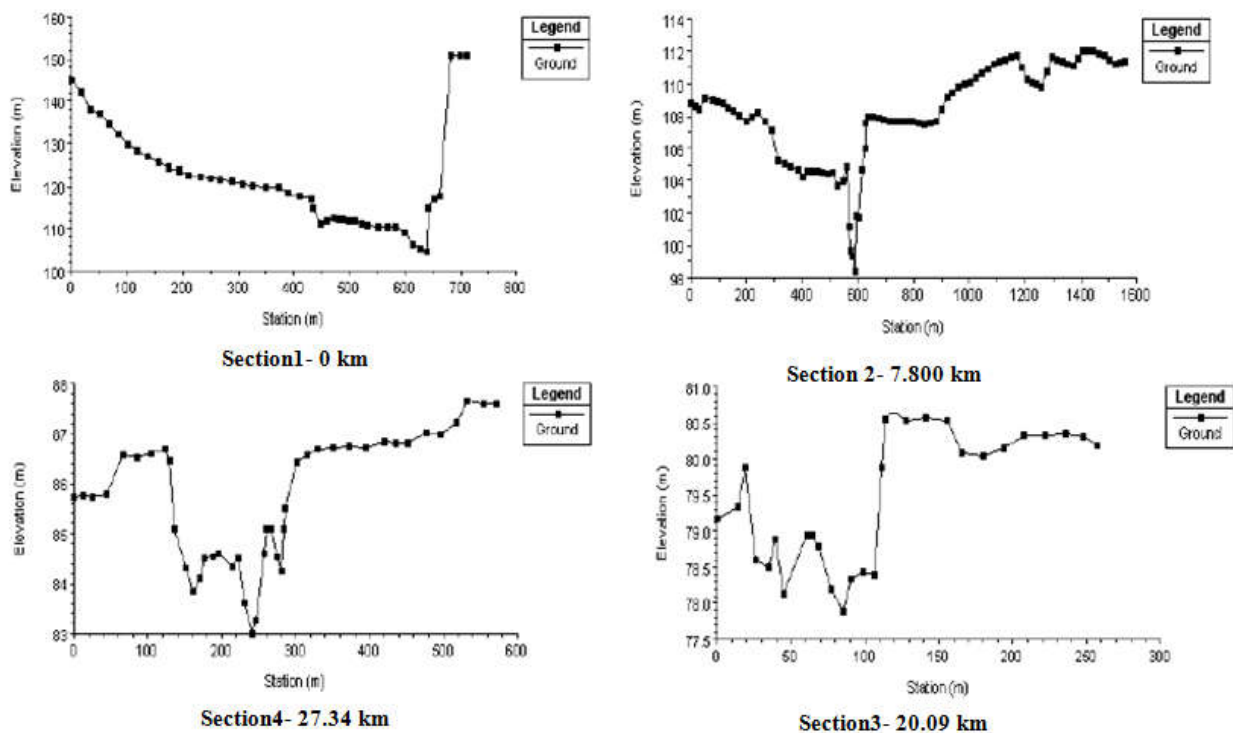


Figure 3. The geometric characteristics of the harvested sections 0 km to 27.34 km

Table 1. Section 1- 0 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/04/21	416.38	189.28	0.7	2.19981	0.000125	0.051927	1E-06	0.000225	0.004
2005/04/22	573.26	207.21	0.91	2.76657	0.000157	0.0652629	1E-06	0.000169	0.004
2005/04/23	540.5	206.31	0.86	2.61984	0.000151	0.0622834	1E-06	0.000177	0.004
2005/04/24	506.29	205.36	0.81	2.46538	0.000145	0.0592068	1E-06	0.000187	0.004
2005/04/25	469.85	204.35	0.77	2.29924	0.000141	0.056383	1E-06	0.000196	0.004

Table 2. Section 2- 7.8 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/04/21	181.99	95.89	1.57	1.898	0.000766	0.1193981	1E-06	9.76E-0.5	0.004
2005/04/28	179.85	92.8	1.56	1.938	0.000737	0.118348	1E-06	9.85E-0.5	0.004
2005/04/29	146.02	61.6	1.36	2.37	0.000424	0.0992761	1E-06	0.000117	0.004
2005/04/30	154.79	75.88	1.38	2.04	0.000539	0.103836	1E-06	0.000112	0.004
2005/04/31	138.77	48.47	1.33	2.863	0.000315	0.09404	1E-06	0.000124	0.004

Table 3. Section 3- 20.09 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/05/01	96.24	60.31	2.9	1.596	0.00328	0.226551	1E-06	5.15 E-0.5	0.0035
2005/05/02	64.18	59.17	2.59	1.085	0.004377	0.2157661	1E-06	5.4 E-0.5	0.0035
2005/05/03	90.05	59.89	2.83	1.504	0.003549	0.2288715	1E-06	5.1 E-0.5	0.0035
2005/05/04	205.6	60.09	2	3.422	0.003409	0.3381971	1E-06	3.45 E-0.5	0.0035
2005/05/05	36.37	54.87	2.39	0.664	0.007203	0.2165525	1E-06	5.38 E-0.5	0.0035

Table 4. Section 4- 27.34 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/05/09	164.45	145.18	1.7	1.133	0.00148	0.1282125	1E-06	9.09E-0.5	0.003
2005/05/10	161.33	142.21	1.704	1.134	0.00149	0.1287475	1E-06	9.06 E-0.5	0.003
2005/05/11	164.03	144.88	1.707	1.132	0.001479	0.1281408	1E-06	9.01 E-0.5	0.003
2005/05/12	143.97	113.25	1.61	1.271	0.001487	0.1361502	1E-06	8.56 E-0.5	0.003
2005/05/13	146.47	113.33	1.625	1.292	0.001481	0.1370015	1E-06	8.51 E-0.5	0.003

Table 5. Section 5- 34.73 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/05/14	170.15	153.01	1.38	1.112	0.001022	0.1055668	1E-06	0.00011	0.002
2005/05/15	191.13	158.33	1.39	1.207	0.000962	0.1067127	1E-06	0.000109	0.002
2005/05/16	179.14	155.33	1.38	1.153	0.00099	0.1058181	1E-06	0.00011	0.002
2005/05/17	243.07	208.68	1.403	1.165	0.000917	0.1023523	1E-06	0.000114	0.002
2005/05/18	174.06	154	1.384	1.13	0.001	0.1052774	1E-06	0.000111	0.002

Table 6. Section 6- 40.83 km

Date	A	P	V	Rh	Sf	U*	N	d'	D ₆₅
2005/05/21	112.3	98.9	1.344	1.135	0.000818	0.0954365	1E-06	0.000122	0.002
2005/05/22	131.67	127.54	1.39	1.032	0.000816	0.090889	1E-06	0.000128	0.002
2005/05/23	122.33	115.84	1.37	1.056	0.000817	0.0919802	1E-06	0.000127	0.002
2005/05/24	113.4	101.47	1.35	1.118	0.000818	0.0946804	1E-06	0.000123	0.002
2005/05/25	93.07	78.96	1.16	1.179	0.000691	0.0893689	1E-06	0.00013	0.002

After the riverbed soil sampling in studies area and applying grading test, we observed that type of riverbed was loamy coarse sand and size of riverbed particles decreased from upstream to downstream. The data collected is shown in Table 1.

Conclusion

The surface of waterway in the entire ranges of study is rough hydrodynamically and the roughness is independent from bottom particle size. The main factor of rough floor hydrodynamically, is steep slope especially in the upstream bed. So can be said that, correction coefficient X is equal to 1 and Δ is equal to D₆₅.

After speed deep profile drawing in each section using collected data and comparison with the velocity predicted profiles by Einstein's equation can be realized high accuracy of this equation for rough surfaces. Velocity gradient predicted by the equation has good agreement with obtained gradient from the study sections, particularly near the bed and high slope of gradient to the vertical line represents the high speed and high turbulence near the ground and consequently instability of bed and this prediction with results of the sediment analysis has confirmed the preferred Yung method in HEC-RAS software (version 4). Prediction of 5-year longitudinal bed profile changes by the Yung pattern was shown in Figure 4. As seen in the upstream interval the dominant phenomenon was erosion and in downstream

interval was not observed any changes (The model outputs is increased for better understanding).

REFERENCES

- Aksoy, H. and Levent Kavvas, M. 2005. "A review of hillslope and watershed scale erosion and sediment transport models". *CATENA*, Vol. 64, No. 2-3, pp. 247-271. doi:10.1016/j.catena.2005.08.008.
- Bhattacharyya, K., Mukhopadhyay, S. and Layek, G. C. 2011. "Slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet". *International Journal of Heat and Mass Transfer*, Vol. 5, No. 1-3, pp. 308-313. doi:10.1016/j.ijheatmasstransfer.2010.09.041.
- Denicol, G.S., Koide, T. and Rischke, D. H. 2010. "Dissipative relativistic fluid dynamics: a new way to derive the equations of motion from kinetic theory". *Phys Rev Lett*, Vol. 105, No. 16, pp.162501. DOI: <http://dx.doi.org/10.1103>.
- Einstein, H. A. and Harder, J. A. 1954. "Velocity distribution and the boundary layer at channel bends". *Eos, Transactions American Geophysical Union*, Vol. 35, No. 1, pp.114-120. DOI: 10.1029/TR035i001p00114.
- Huai, W. X., Han, J., Zeng, Y. H., An, X. and Qian, Z. D. 2009. "Velocity distribution of flow with submerged flexible vegetations based on mixing-length approach". *Applied Mathematics and Mechanics*, Vol. 30, No. 3, pp. 343-351. DOI: 10.1007/s10483-009-0308-1.
- Wu, X. and Moin, P. 2009. "Direct numerical simulation of turbulence in a nominally zero-pressure-gradient flat-plate boundary layer". *Journal of Fluid Mechanics*, Vol. 630, pp. 5- 41. DOI: <http://dx.doi.org/10.1017/S0022112009006624>.
