



RESEARCH ARTICLE

FUZZY SYNCHRONIZATION OF THE 3D CHAOTIC SYSTEMS USING BELBIC

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ABSTRACT

This paper presents fuzzy model-based designs for synchronization of new chaotic system. The T-S fuzzy models for new chaotic systems are exactly derived. Then utilizing an intelligent controller which based on brain emotional learning (BELBIC), this fuzzy chaotic system is synchronized. Numerical simulation results are presented to show the effectiveness of the proposed method.

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INTRODUCTION

Control of complex irregular dynamics has evolved as one of the central issues in applied nonlinear science during the last decade. Nowadays the notion of chaos control involving stabilization of unstable periodic or stationary states in nonlinear dynamic systems has been extended to a much wider class of problems. Since the discovery of chaos synchronization introduced in (Carroll, 1990), there have been tremendous interests in studying the synchronization of chaotic systems. Recently, much research on the fuzzy model-based designs to stability and synchronization for chaotic systems have been carried out based on Takagi-Sugeno (T-S) fuzzy models (Park et al., 2002; Lian et al., 2001). In (Yan-Wu Wang et al., 2003), a fuzzy model-based designs for Chen's chaotic stability and synchronization have been proposed. Based on the fuzzy hyper chaotic models, simpler fuzzy controllers have been designed for synchronizing hyper chaotic systems in (Hongbin Zhang, 2005). In this work, utilizing BELBIC model introduced in (Ali Reza Mehrabian et al., 2006; Saeed Jafarzadeh et al., 2008), we will design an intelligent controller for synchronization of two new 3D

chaotic systems (Sara Dadras and Hamid Reza Momeni, 2009). Simulation results depicts that this proposed controller can synchronize these chaotic systems. The rest of the paper is organized as follows. In Section 2, the Brain Emotional Learning Based Intelligent Controller (BELBIC) is described. In Section 3, the T-S fuzzy models will be presented for a new 3D chaotic system (Sara Dadras and Hamid Reza Momeni, 2009). In Section 4, synchronization between two new fuzzy chaotic system is achieved by BELBIC is described. Finally, Section 5 provides, conclusion of this work.

**Brain Learning Based Intelligent Controller (BELBIC)**

In this method, emotional factors like excitement and anxiety are the roots of learning. Here, the roots of anxiety are assumed as some stimulants and the control system should react in the way that reduces the system anxiety that is caused by these stimulants. The Brain Emotional Learning (BEL) is divided into two parts, very roughly corresponding to the amygdala and the orbitofrontal cortex, respectively. The amygdaloid part receives inputs from the thalamus and from cortical areas, while the orbital part receives inputs from the cortical areas and the amygdala only. The system also receives reinforcing (REW) signal. The emotional learning model in amygdala and orbitofrontal cortex is illustrated in Figure 1.

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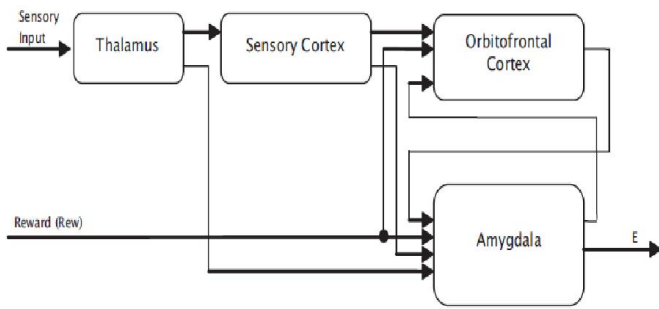


Figure 1. Scheme of BELBIC structure (Lucas et al., 2004)

BELBIC has some input sensors that can be chosen by designer. Each input sensor has two different states that can be described as.

$$A_i = s_i v_i$$

$$O_i = s_i w_i \dots \dots \dots (1)$$

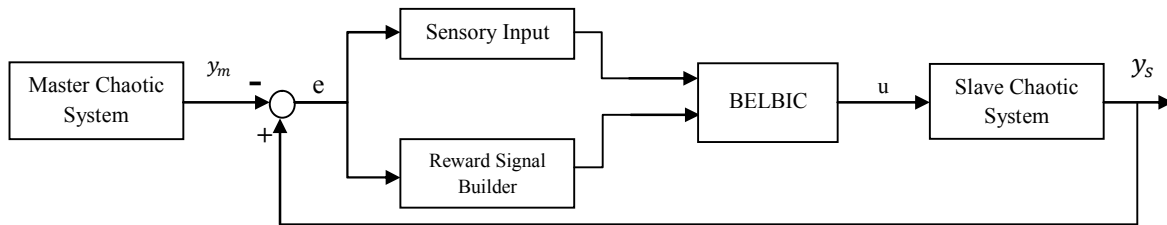


Figure 2. Control system configuration using BELBIC

In which,  $s_i$  is the input sensor and  $v_i$  are two states that are depended on input sensor. Index  $i$  represents the  $i^{th}$  sensor and its related states. These two will be updated by following equations (Saeed Jafarzadeh et al., 2008).

$$\Delta v_i = \alpha s_i \max(0, rew - \sum A_i)$$

$$\Delta w_i = \beta s_i (rew - \sum A_i - \sum O_i - \max(s_i)) \dots \dots \dots (2)$$

In which  $\alpha, \beta$  are training coefficients and  $rew$  is reward signal. Amygdala acts as an actuator and orbitofrontal cortex acts as a preventer. Therefore the control signal of BELBIC is.

$$u = \sum A_i - \sum O_i \dots \dots \dots (3)$$

This paper uses the continuous form of BELBIC. In continuous form the BELBIC states are updated by following equations.

$$\dot{v}_i = \alpha s_i (rew - A_i)$$

$$\dot{w}_i = \beta s_i (rew + s_i + O_i - A_i) \dots \dots \dots (4)$$

A BELBIC controller has to be designed for synchronization two chaotic systems. For traction force sensory inputs considered.

$$s_i = e_i \dots \dots \dots (5)$$

$e_i$  is error between master system and slave system. The structure of the control system is illustrated in Figure 2. Reward signal will be obtained reward function. This function

has a great role in BELBIC. Designer must define a reward function that has its maximum values in the most desired regions. In this work, the reward function is chosen as a linear function of system error.

$$Rew_i = k_{1i} e_i + k_{2i} \dots \dots \dots (6)$$

$k_{1i}$  and  $k_{2i}$  are positive parameters of reward function. The reward function for this BELBIC controller is as Figure 3.

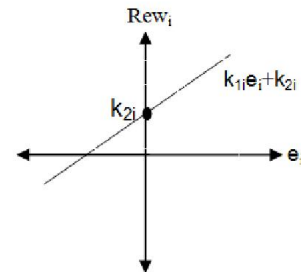


Figure 3. Reward Function

### T-S Fuzzy Model of New 3D Chaotic System

The T-S fuzzy dynamic model originates from Takagi and Sugeno, (1985), which is described by fuzzy IF-THEN rules where the consequent parts represent local linear models. Consider a continuous-time nonlinear dynamic equation as

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \dots \dots \dots (7)$$

Then the T-S fuzzy model is composed of the following rules:

Rule  $i$ : if  $z_1(t)$  is  $M_{i1}$  and  $\dots$  and  $z_p(t)$  is  $M_{ip}$  then  $\dot{x}(t) = A_i x(t) + B u(t)$ ,  $i = 1, 2, \dots, r$ ..... (8)

Where  $Z_1(t) \sim Z_p(t)$  are the premise variables which would consist of the states of the system,  $M_{ij} (j = 1, 2, \dots, p)$  are fuzzy sets,  $r$  is the number of fuzzy rules,  $A_i, B$  are system matrices with appropriate dimensions. The final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^r h_i(z(t)) (A_i x(t) + B u(t)) \dots \dots \dots (9)$$

Where,

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^r w_i(z(t))}, w_i(z(t)) = \prod_{j=1}^p M_{ij}(z_j(t)) \dots \dots (10)$$

Recently, Sara Dadras and Hamid Reza Momeni constructed the 3D autonomous chaotic system (Sara Dadras and Hamid Reza Momeni, 2009). The system is described by.

$$\begin{aligned} \dot{x} &= y - ax + byz \\ \dot{y} &= cy - xz + z \\ \dot{z} &= dxy - hz \end{aligned} \quad (11)$$

Where  $a, b, c, d, h$  are positive constants and  $x, y, z$  are variables of the system, when  $a = 3, b = 2.7, c = 4.7, d = 2, h = 9$ , the system (11) is chaotic. See Figure 4.

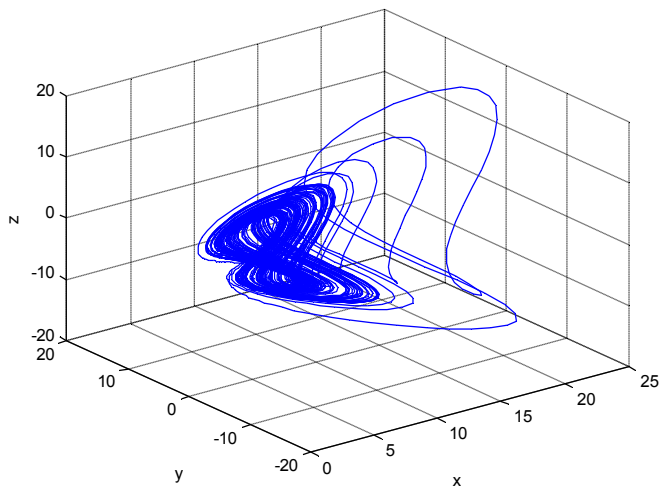


Figure 4. Strange attractors system (11)

Assume that  $x \in [-g, g], y \in [-g, g]$  and  $g > 0$ , then system (11) can be exactly represented by T-S fuzzy model as following:

- Rule 1: if  $x$  is  $M_1$  and  $y$  is  $N_1$  then  $\dot{X}(t) = A_1X(t)$
- Rule 2: if  $x$  is  $M_1$  and  $y$  is  $N_2$  then  $\dot{X}(t) = A_2X(t)$
- Rule 3: if  $x$  is  $M_2$  and  $y$  is  $N_1$  then  $\dot{X}(t) = A_3X(t)$
- Rule 4: if  $x$  is  $M_2$  and  $y$  is  $N_2$  then  $\dot{X}(t) = A_4X(t)$ .....(12)

Here,

$$\begin{aligned} X &= [x \quad y \quad z]^T \\ M_1(x) &= \frac{1}{2} \left( 1 + \frac{x}{g} \right), M_2(x) = \frac{1}{2} \left( 1 - \frac{x}{g} \right) \\ N_1(y) &= \frac{1}{2} \left( 1 + \frac{y}{g} \right), N_2(y) = \frac{1}{2} \left( 1 - \frac{y}{g} \right) \end{aligned} \quad (13)$$

And states matrixes are:

$$\begin{aligned} A_1 &= \begin{bmatrix} -a & 1 & bg \\ 0 & c & 1-g \\ 0 & dg & -h \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -a & 1 & -bg \\ 0 & c & 1-g \\ 0 & dg & -h \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -a & 1 & bg \\ 0 & c & 1+g \\ 0 & -dg & -h \end{bmatrix}, \\ A_4 &= \begin{bmatrix} -a & 1 & -bg \\ 0 & c & 1+g \\ 0 & -dg & -h \end{bmatrix} \end{aligned} \quad (14)$$

That  $g = 30$  So the final output of the fuzzy system (12) is inferred as follows and the chaotic behavior is shown in Figure 5.

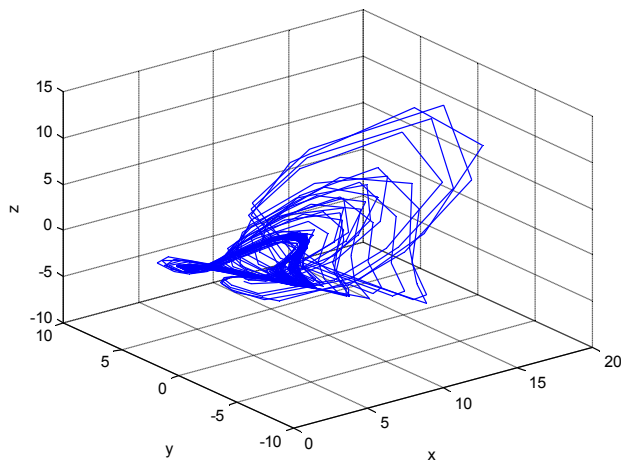


Figure 5. Chaotic behavior of fuzzy system (12)

$$\dot{X}(t) = \sum_{i=1}^4 M_i(x)N_i(y)A_i X(t) \dots \dots \dots (15)$$

To support our analysis to be carried out in the following sections, some existing results are needed.

### Fuzzy synchronization of new chaotic system

In order to achieve the behavior of the synchronization between two new fuzzy chaotic system by using the BELBIC, suppose the drive system (12) and the slave system takes the following from.

- Rule 1: if  $\tilde{x}$  is  $M_1$  and  $\tilde{y}$  is  $N_1$  then  $\dot{\tilde{X}}(t) = A_1\tilde{X}(t) + Bu(t)$
- Rule 2: if  $\tilde{x}$  is  $M_1$  and  $\tilde{y}$  is  $N_2$  then  $\dot{\tilde{X}}(t) = A_2\tilde{X}(t) + Bu(t)$
- Rule 3: if  $\tilde{x}$  is  $M_2$  and  $\tilde{y}$  is  $N_1$  then  $\dot{\tilde{X}}(t) = A_3\tilde{X}(t) + Bu(t)$
- Rule 4: if  $\tilde{x}$  is  $M_2$  and  $\tilde{y}$  is  $N_2$  then  $\dot{\tilde{X}}(t) = A_4\tilde{X}(t) + Bu(t)$ .. (16)

Where,

$$\begin{aligned} \tilde{X} &= [\tilde{x} \quad \tilde{y} \quad \tilde{z}]^T, \\ u(t) &= [u_1(t) \quad u_2(t) \quad u_3(t)]^T, \\ B &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (17)$$

Then the defuzzification process is given as

$$\dot{\tilde{X}}(t) = \sum_{i=1}^4 M_i(\tilde{x})N_i(\tilde{y}) (A_i\tilde{X}(t) + Bu(t)) \dots \dots \dots (18)$$

Assume that  $e(t) = [e_x(t) \quad e_y(t) \quad e_z(t)] = \tilde{X}(t) - X(t)$ , we obtain

$$\dot{e}(t) = \sum_{i=1}^4 M_i(x)N_i(y)A_i X(t) - \sum_{i=1}^4 M_i(\tilde{x})N_i(\tilde{y}) (A_i\tilde{X}(t) + Bu(t)) \dots \dots \dots (19)$$

Thus, the errors system (19) to be controlled with control inputs  $u_1(t), u_2(t)$  and  $u_3(t)$  as functions of error states  $e_x, e_y$  and  $e_z$ . When system (19) is stabilized by control inputs  $u_1(t), u_2(t)$  and  $u_3(t)$ ,  $e_x, e_y$  and  $e_z$  will converge to zeroes as time  $t$  tends to infinity. Which implies that system (15) and (18) are synchronized.

To achieve this purpose, Input sensory of BELBIC is chosen as (20).

$$\begin{cases} s_1 = e_x \\ s_2 = e_y \\ s_3 = e_z \end{cases} \dots\dots\dots(20)$$

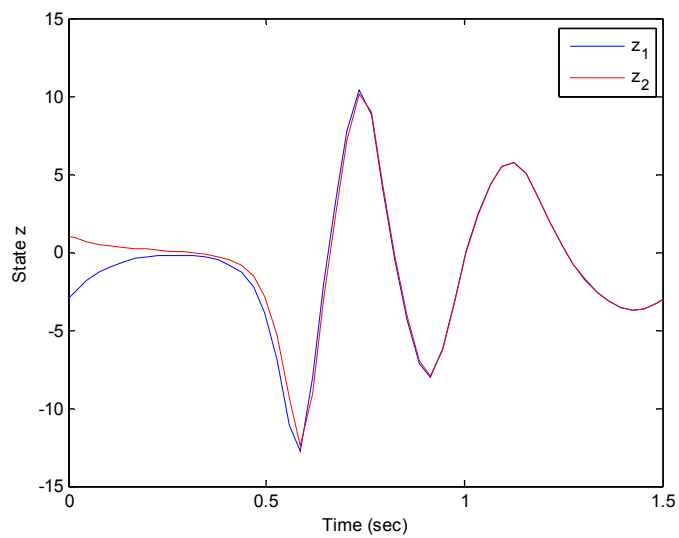
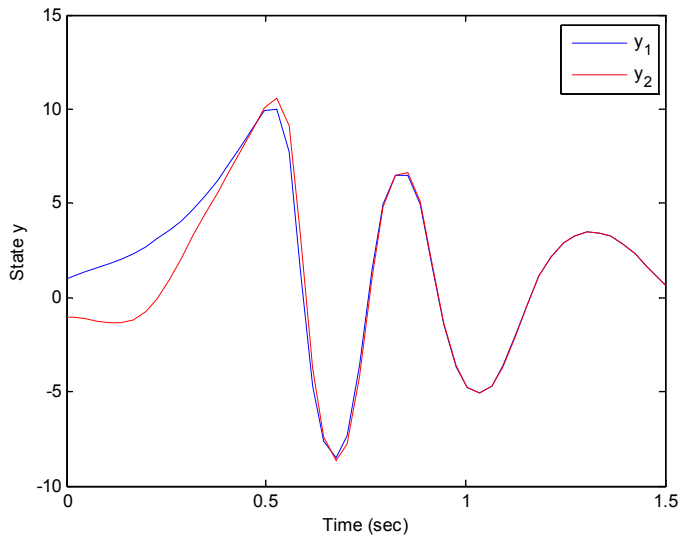
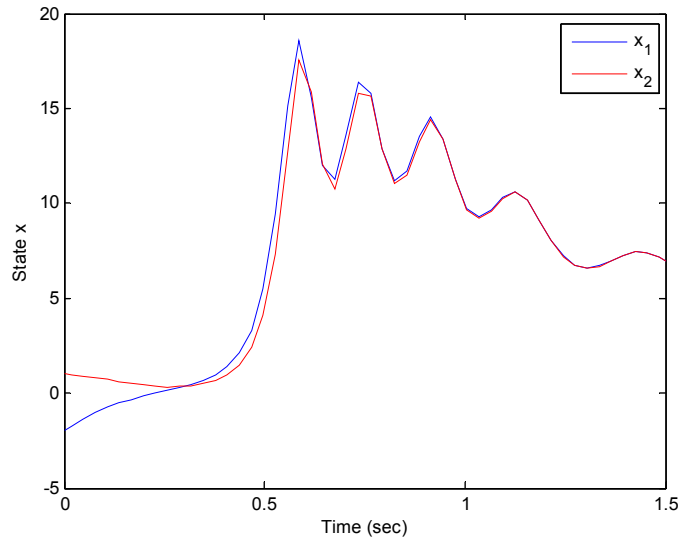


Figure 6. The time response of signals  $(x, y, z)$  for drive system (7) and response system (17)

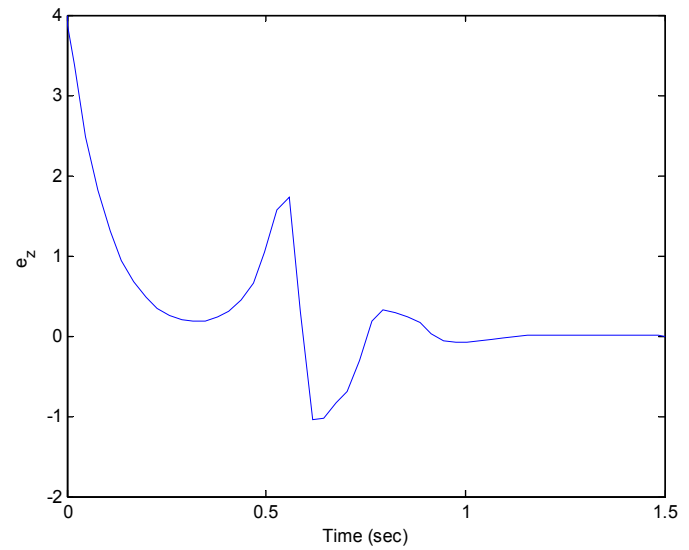
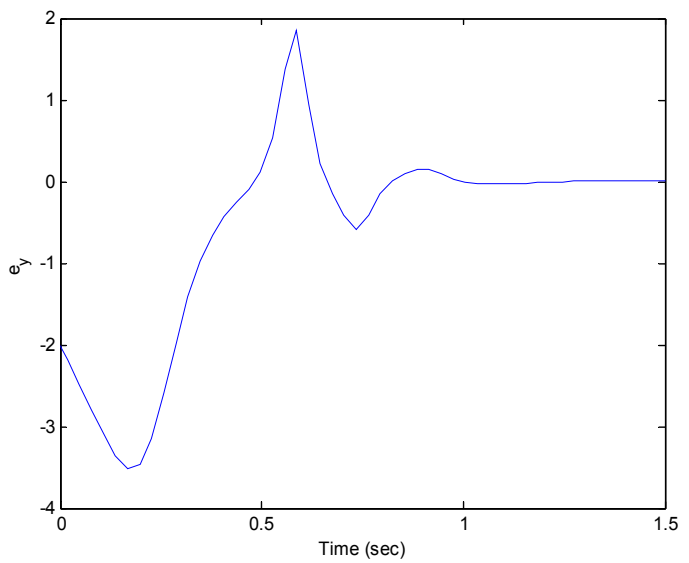
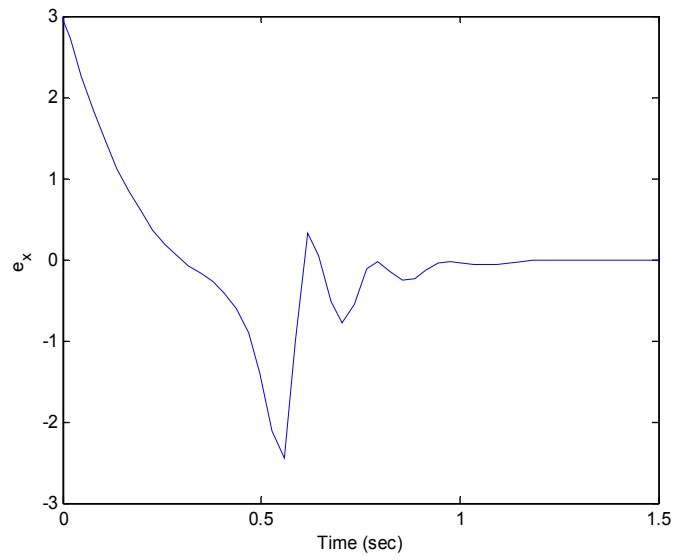
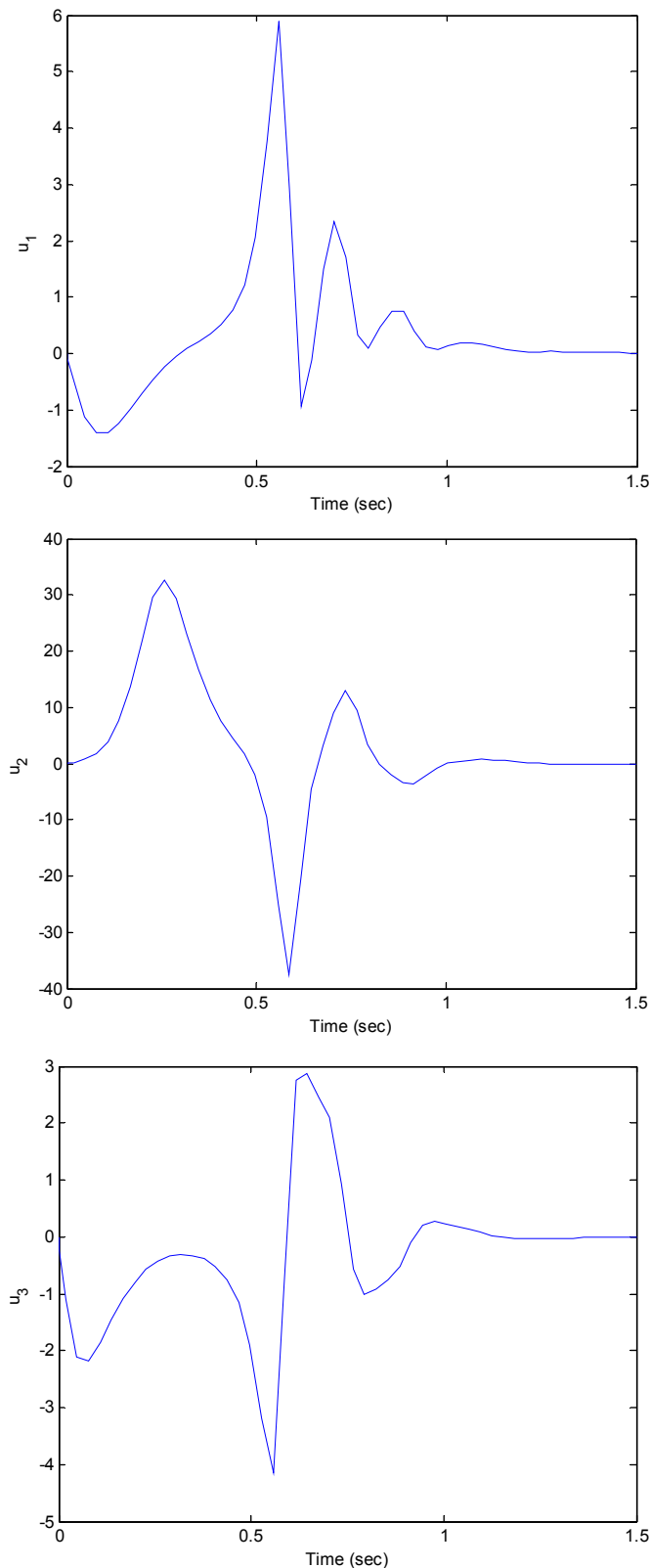


Figure 7. Synchronization errors  $(e_x, e_y, e_z)$  in drive system (7) and response system (17)



**Figure 8. The time response of the control inputs ( $u_1, u_2, u_3$ ) for drive system (7) and response system (17)**

The reward function's parameters for the BELBIC controller are as follows.

$$\begin{cases} rew_1 = k_1 e_x + k_2 \\ rew_2 = k_3 e_y + k_4 \\ rew_3 = k_5 e_z + k_6 \end{cases} \quad (21)$$

We select the gains of reward signals (21)  $k_i = 1, i = 1, \dots, 6$ . The initial values of the drive and response systems are  $x_1(0) = -2, y_1(0) = 1, z_1(0) = -3$  and  $x_2(0) = 1, y_2(0) = -1, z_2(0) = 1$  respectively. The time response of  $x, y, z$  states for drive system (15) and the response system (18) shown in Figure 6. Synchronization errors ( $e_x, e_y, e_z$ ) in the new fuzzy chaotic systems shown in Figure 7. The time response of the control inputs ( $u_1, u_2, u_3$ ) for the synchronization new fuzzy chaotic systems shown in Figure 8.

**Conclusions**

In this paper, a fuzzy model-based designs for new 3D chaotic stability and synchronization have been proposed. Based on the fuzzy chaotic models, simpler fuzzy controllers have been designed for synchronizing new chaotic systems via BELBIC. Finally, numerical simulation was given to verify the effectiveness of the proposed controllers.

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