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# MEASURES OF MULTIDIMENSIONAL BETWEEN-GROUP INEQUALITY

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**ABSTRACT**

This study is an attempt to develop two multidimensional between-group inequality measures by reviewing two multidimensional interpersonal inequality measures. These measures of multidimensional between-group inequality are characterized and applied on Indian data to assess between-group inequality in household monthly per capita consumer expenditure and educational achievement across fourteen major states of India.



## INTRODUCTION

Usually interpersonal and between-group inequality is assessed in the distribution of one well-being indicator, such as income, educational achievement and health status. There is no problem associated with the analysis. However, while one assesses between-group inequality in multidimensional space, then some problems arise. Multidimensional between-group inequality can be viewed as a combination of two kinds of effects. One is due to the degree of between-group inequality in each dimension and the other is due to the possible intensification because of the high degree of correlation between-group inequalities in different dimensions (List, 1999). Therefore, multidimensional inequality can be viewed as a combination of two kinds of effects. One is due to the degree of inequality in each dimension and the other is due to the possible intensification because of the high degree of correlation between inequalities in different dimensions (List, 1999)[1]. Kolm (1977) has developed the dominance criteria by multidimensional generalization of the Pigou-Dalton transfer principle.

The multidimensional inequality measures which satisfy these dominance criteria enable us to compare the degrees of multidimensional inequality between two or more distributions[2]. Some scholars have developed multidimensional measure of inequality using an aggregation function (Maasoumi, 1986), which converts the multidimensional distribution into a distribution of utilities. The multidimensional inequality index is then obtained by applying a univariate index of inequality to this distribution. Later on, Tsui (1995) generalized the univariate Atkinson-Kolm-Sen approach. The measures developed by Tsui (1995) satisfy only those dominance criteria which are developed by the multidimensional generalization of the Pigou-Dalton transfer principle. The dominance criterion, which was developed from the Pigou-Dalton principle of transfer, cannot develop a measure of multidimensional inequality which would incorporate the idea of cross-correlation[3]. For this

[1] The relative advantage of a group in one dimension can weaken the relative disadvantages of that group in other dimensions, which can bring down the degree of total inequality derived by simple additions of the degrees of inequality across dimensions of the multidimensional distribution. On the contrary, consistent relative disadvantages of a certain group across dimensions can intensify the overall degrees multidimensional inequality which is over and above simple addition of the degrees of inequality in different dimensions of the multidimensional distribution.

[2] The dominance criteria are discussed in Appendix I.

[3] Let there be three multidimensional distributional matrices for two groups and two attributes, A, B and C, where groups are measured along rows and attributes are measured along columns, and the cell elements of the matrices are the group averages of the attributes.

$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 4 \\ 5 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 8 & 1 \\ 1 & 8 \end{bmatrix}$

In the distribution A the first group is advantaged in the first dimension and the second group is advantaged in the second dimension. In distribution B, the first group is disadvantaged in both dimensions and the second group is advantaged in all dimensions. In C, like in A, the first group is advantaged in the first and the second person is disadvantaged in the second dimension.

reason, Tsui (1999) and List (1999) have introduced another dominance criterion, i.e. *correlation increasing majorization*, which can take care of the systematic cross-correlation between inequalities in different dimensions. They have developed some measures of multidimensional interpersonal inequality, which satisfy the dominance criteria developed from the multidimensional generalization of transfer principle as well as the *correlation increasing majorization*. The major objective of this study is to develop two multidimensional between-group inequality measures by reviewing the existing multidimensional interpersonal inequality measures and following the method of Maasoumi (1986), then characterize these measures by developing a set of axioms relevant for characterizing the multidimensional between-group inequality. Then apply the developed multidimensional between-group inequality on Indian data. One of these measures does not satisfy measure *correlation increasing majorization* but other satisfies it[4]. The rest of this paper is structured as follows. Section two explains two multidimensional between-group inequality measures. Section three describes the empirical illustration of these newly developed measures. The fifth section concludes.

**Multidimensional between-group inequality indices:** The first multidimensional between-group inequality measure is developed from the AKS index developed in Tsui (1995). The second measure is developed by converting the Generalized Gini index (List, 1999) to adapt it to suit between-group inequality.

**Group analogue of multidimensional AKS measure:** First we develop the group analogue of the AKS index of multidimensional interpersonal inequality following Tsui (1995). Let there be K well-defined groups and M attributes, and the social welfare/evaluation function (SEF) be continuous, increasing, concave and additively separable. Assume that all individuals included in a group are alike. This assumption enables us to view the groups as individuals, and we can derive the group analogue of AKS index, which has the following form:

$$I^{AKS} = 1 - \left(\sum_{i=1}^{K} \lambda_i \left(\prod_{j=1}^{M} \left(\frac{\mu_{ij}}{\mu_j}\right)^{r_j}\right)\right)^{\frac{1}{\sum r_j}} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Where $\lambda_i$ is the population share of the i-th group, $\mu_{ij}$ is the mean of the j-th attribute for the i-th group and $\mu_j$ is the mean value of the j-th attribute.

Index (1) satisfies the *normalization, within-group anonymity*, *between-group anonymity*, *scale invariance*, *total population size, group replication invariance,* u*niform majorization* and *uniform Pigou-Dalton majorization* principles except *population composition invariance* principle and *correlation increasing majorization* criterion.

However, C can be obtained by regressive transfer from the first to the second group in the first dimension and the second to the first group in the second dimension. Inequality in C is more than in A. Inequality in A is less than in B. However, we cannot compare B and C unless we are able to assess the relative strengths of inequalities in different dimensions and systematic cross-correlation between inequalities in different dimensions.

[4]The axiomatic properties and the dominance criteria are discussed in Appendix I.

**Group analogue of multidimensional Generalized Gini Index (GGI):** Following the method of formation of multidimensional Generalized Gini (List, 1999), we develop its group analogue form, by assuming that the individuals in a groups are alike, i.e. we can view groups as individuals. If there are K groups and M attributes in the multidimensional distribution, then the group analogue of Generalized Gini Index (GGI) has the following form:

$$I^{GGI} = 1 - \eta + \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Where $\eta = \left(\frac{1}{M}\right)\sum_{i=1}^{K} \lambda_i . \left(\sum_{j=1}^{M}\left(\frac{\mu_{ij}}{\mu_j}\right)^{r_j}\right)$ and

$$\theta = \left(\frac{1}{MK(K-1)}\right)\sum_{\substack{i=1 \\ i>j}}^{K}\sum_{l=1}^{K}\left|\left(\sum_{j=1}^{M}\left(\frac{\mu_{ij}}{\mu_j}\right)^{r_j}\right) - \left(\sum_{j=1}^{M}\left(\frac{\mu_{lj}}{\mu_j}\right)^{r_j}\right)\right| . \lambda_i . \lambda_l$$

Where $\lambda_i$ and $\lambda_l$ are the population shares of the i-th and l-th groups; $\mu_{ij}$ and $\mu_j$ are the average value of the j-th attribute for i-th group and the average value of the j-th attribute; and $r_j$ is a parameter representing the concavity of the function from which we this index has been developed. The minimum value of the index $I^{GGI}$ is 0, and its value rises with the rise in between-group inequality.

This index satisfies *within-group anonymity*, *scale invariance*, *total population size* invariance and *group replication invariance* principles. However, it does not satisfy *population composition invariance* and *between-group anonymity* principles. Moreover, this index also satisfies the *uniform majorization*, *uniform Pigou-Dalton majorization* and *correlation increasing majorization* criteria. Therefore, this index can order larger set of multidimensional distributions according to the degrees of between-group inequality as well as on the basis of the systematic cross-correlation between between-group inequalities in different dimensions of the multidimensional distributions.

## Empirical illustration

**Data source and methods:** We use the data set from Indian Human Development Survey (IHDS; 2004-05), conducted by National Council of Applied Economic Research (NCAER), New Delhi, India, in collaboration with the University of Maryland to compute the value of multidimensional between-group inequality in India. The data were collected on the basis of stratified random sampling procedure, which covered 41554 households and 215784 individuals in rural and urban areas of India. Moreover, we include the persons aged above 24 years in the sample. Therefore, finally the restricted sample came down to 91214 individuals (59429 in rural and 31785 in urban). We take two attributes - monthly per capita consumer expenditure ('mpce') and educational achievements (measured in terms of 'years of schooling') of the individuals, and compute the multidimensional inequality among four social groups, namely 'Others', other backward caste (OBCs), scheduled Castes (SCs), and Scheduled Tribes (STs) in rural and urban areas across fourteen major states of India. The relative and intermediate forms of two aforementioned multidimensional between-group inequality measures are also used in the analysis. To compute the group analogue of the multidimensional AKS index ($I^{AKS}$) and group analogue of the multidimensional GGI ($I^{GGI}$), we take the value of the inequality aversion parameter $r_j = 0.5$, $\forall j = 1, 2, \ldots, M$.

**Multidimensional between-group inequality in major states of India:** Tables 1 and 2 report the average values of household monthly per capita consumer expenditure and average years of schooling of four social groups across fourteen major states of India.

Table 3 presents the computed values of the group analogue of the relative AKS and Generalized Gini indices. We find a systematic difference among the states of India. Rural areas of Bihar ($I^{AKS}$ = 0.157), Orissa ($I^{AKS}$ = 0.129), Uttar Pradesh ($I^{AKS}$ = 0.101) and Madhya Pradesh ($I^{AKS}$ = 0.102) respectively, have the first, second, third and fourth highest incidences of Multidimensional inequality among the social groups is the least in Tamil Nadu ($I^{AKS}$ = 0.029) and the

**Table 1. Average 'mpce' of the social groups in India and in its fourteen major states (in Rupees)**

| States | Rural (A) | | | | | Urban (B) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Others | OBC | SC | ST | Total | Others | OBC | SC | ST | Total |
| Andhra Pradesh | 976.83 | 778.24 | 693.67 | 730.8 | 845.92 | 1463.4 | 1150.83 | 1213.25 | 1089.06 | 1310.21 |
| Assam | 1127.55 | 646.03 | 708.04 | 583.09 | 782.77 | 1352.21 | 1063.28 | 843.48 | 1087.03 | 1186.05 |
| Bihar | 732.56 | 613.06 | 505.32 | 389.52 | 587.79 | 1408.45 | 821.96 | 602.06 | 274 | 1007.33 |
| Gujarat | 863.2 | 736.72 | 727.14 | 517.27 | 795.25 | 1558.88 | 950.13 | 974.65 | 862.61 | 1275.09 |
| Karnataka | 865.26 | 776.92 | 499.79 | 617.9 | 788.05 | 1613.7 | 1373.5 | 979.09 | 929.1 | 1420.94 |
| Kerala | 1069.07 | 926.08 | 651.13 | 684.41 | 969.13 | 1387.09 | 912.38 | 739.59 | 751 | 1109.66 |
| Madhya Pradesh | 637.62 | 518.95 | 455.34 | 268.48 | 583.52 | 1237.24 | 776.36 | 671.73 | 618.85 | 993.95 |
| Maharashtra | 691.53 | 702.07 | 597.93 | 443.02 | 784.28 | 1151.72 | 996.5 | 831.58 | 985 | 1107.25 |
| Orissa | 638.62 | 485.72 | 372.49 | 324.85 | 573.21 | 1595.04 | 1018.78 | 681.9 | 546.35 | 941.18 |
| Punjab | 1183.68 | 988.16 | 781.54 | 855.17 | 998.34 | 1381.53 | 1077.36 | 1044.82 | 1090 | 1253.87 |
| Rajasthan | 745.56 | 689.53 | 543.81 | 541.32 | 689.26 | 1487.22 | 877.4 | 706.53 | 927.09 | 1139.53 |
| Tamil Nadu | 958.79 | 819.69 | 640.8 | 479.16 | 792.15 | 1549.49 | 1118.09 | 858.11 | 534.57 | 1134.43 |
| Uttar Pradesh | 878.04 | 701.71 | 473.64 | 431.96 | 639.78 | 1470.1 | 929.57 | 802.16 | 1110.48 | 907.83 |
| West Bengal | 878.57 | 790.75 | 573.7 | 453.38 | 730.91 | 1442.38 | 1137.01 | 795.47 | 1161.91 | 1283.91 |
| All India | 852.99 | 760.88 | 568.18 | 439.16 | 749.4 | 1447.58 | 1058.88 | 883.71 | 975.76 | 1205.67 |

*Source: Indian Human Development Survey Data, 2004-05.*

**Table 2. Average years of schooling of the social groups in India and its fourteen major states**

| States | Rural (A) | | | | | Urban (B) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Others | OBC | SC | ST | Total | Others | OBC | SC | ST | Total |
| Andhra Pradesh | 4.68 | 3.03 | 2.94 | 2.13 | 3.88 | 6.79 | 5.12 | 5.15 | 5.29 | 5.97 |
| Assam | 6.9 | 5.37 | 4.92 | 4.43 | 5.27 | 7.76 | 8.04 | 7.03 | 6.27 | 7.87 |
| Bihar | 4.87 | 2.43 | 1.52 | 0.43 | 2.91 | 7.46 | 5.25 | 3.15 | 6.66 | 6.31 |
| Gujarat | 4.72 | 3.69 | 3.85 | 2.48 | 4.12 | 8.33 | 5.25 | 5.53 | 5.28 | 6.92 |
| Karnataka | 5.05 | 4.37 | 2.85 | 3.33 | 4.19 | 8.33 | 6.51 | 5.07 | 4.91 | 6.84 |
| Kerala | 7.71 | 7.07 | 5.65 | 5.17 | 7.26 | 8.95 | 7.53 | 7.1 | 7.05 | 8.01 |
| Madhya Pradesh | 4.51 | 3.08 | 2.48 | 1.7 | 3.27 | 8.1 | 5.06 | 3.8 | 3.45 | 7.03 |
| Maharashtra | 4.92 | 4.81 | 4.29 | 3.67 | 4.89 | 7.43 | 7.05 | 5.47 | 4.83 | 6.96 |
| Orissa | 4.88 | 4.11 | 3.03 | 2.22 | 4.18 | 8.5 | 6.59 | 3.78 | 4.78 | 7.25 |
| Punjab | 4.78 | 4.58 | 3.47 | 3.25 | 4.21 | 8.75 | 6.43 | 6.1 | 5.4 | 7.46 |
| Rajasthan | 3.83 | 3.13 | 2.3 | 2.05 | 3.15 | 7.52 | 4.62 | 3.54 | 3.79 | 6.37 |
| Tamil Nadu | 5.43 | 5.12 | 4.1 | 3.73 | 4.02 | 8.27 | 6.51 | 5.07 | 3.85 | 7.24 |
| Uttar Pradesh | 4.96 | 3.17 | 2.52 | 1.84 | 3.73 | 7.49 | 5.22 | 3.69 | 7.24 | 6.78 |
| West Bengal | 5.38 | 4.69 | 3.06 | 1.6 | 3.94 | 8.4 | 6.64 | 5.03 | 5.31 | 7.56 |
| All India | 5.06 | 3.71 | 2.71 | 2.71 | 4.09 | 7.92 | 6.09 | 4.95 | 5.37 | 7.28 |

Source: Indian Human Development Survey Data, 2004-05.

**Table 3. Multidimensional inequality among the social groups in rural areas**

| States | Rural | | Urban | |
|---|---|---|---|---|
| | $I^{AKS}$ $r_j(\forall j) = 0.5$ (2) | $I^{GGI}$ $r_j(\forall j) = 0.5$ (4) | $I^{GGI}$ $r_j(\forall j) = 0.5$ (4) | $I_C^{GGI}$ $r_j(\forall j) = 0.5$ (5) |
| Andhra Pradesh | 0.075 (6) | 0.069(8) | 0.0813 (10) | 0.084(11) |
| Assam | 0.078 (5) | 0.065(9) | 0.105 (5) | 0.067(14) |
| Bihar | 0.157 (1) | 0.174(1) | 0.169 (2) | 0.175 (2) |
| Gujarat | 0.041(12) | 0.058(11) | 0.058(11) | 0.069(12) |
| Karnataka | 0.049 (9) | 0.083(6) | 0.074 (9) | 0.102(7) |
| Kerala | 0.045(10) | 0.052(12) | 0.042 (14) | 0.068 (13) |
| Madhya Pradesh | 0.102 (4) | 0.133(3) | 0.184 (1) | 0.201 (1) |
| Maharashtra | 0.043(11) | 0.044(13) | 0.076 (8) | 0.097(9) |
| Orissa | 0.129 (2) | 0.148(2) | 0.132(4) | 0.157(4) |
| Punjab | 0.031(13) | 0.038(14) | 0.091 (7) | 0.01(8) |
| Rajasthan | 0.069 (7) | 0.098(5) | 0.098(6) | 0.109(6) |
| Tamil Nadu | 0.029 (14) | 0.062(10) | 0.044(13) | 0.135(5) |
| Uttar Pradesh | 0.101 (3) | 0.119(4) | 0.138 (3) | 0.159(3) |
| West Bengal | 0.051(8) | 0.078(7) | 0.055(12) | 0.09(10) |
| All India | 0.106 | 0.117 | 0.116 | 0.185 |

*Note: We take $\alpha_j = 100$ for monthly per capita consumption expenditure and $\alpha_j = 1$ for years of schooling.*
*Source: Indian Human Development Survey Data, 2004-05.*

second lowest position is occupied by Punjab ($I^{AKS}$ =0.031), which are relatively richer states. Likewise, in the urban areas relative poorer states, Madhya Pradesh ($I^{AKS}$ = 0.184), Bihar ($I^{AKS}$ = 0.169), Uttar Pradesh ($I^{AKS}$ = 0.138) and Orissa ($I^{AKS}$ = 0.132) respectively, have the first, second, third and fourth highest incidences of multidimensional inequality among the social groups in urban India. Kerala and Tamil Nadu occupy the bottom and next to the bottom positions in terms of the relative group analogue of the AKS index. Multidimensional inequality among the social groups is the least in Tamil Nadu ($I^{AKS}$ = 0.029) and the second lowest position is occupied by Punjab ($I^{AKS}$ =0.031), which are relatively richer states. Likewise, in the urban areas relative poorer states, Madhya Pradesh ($I^{AKS}$ = 0.184), Bihar ($I^{AKS}$ = 0.169), Uttar Pradesh ($I^{AKS}$ = 0.138) and Orissa ($I^{AKS}$ = 0.132) respectively, have the first, second, third and fourth highest incidences of multidimensional inequality among the social groups in urban India. Kerala and Tamil Nadu occupy the bottom and next to the bottom positions in terms of the relative group analogue of the AKS index.

It is observed that in rural Andhra Pradesh, Assam, Kerala, Maharashtra, and Punjab, some of the groups are advantaged in one dimension and relatively disadvantaged in other. So, the systematic cross-correlation between between-group inequalities in different dimensions is not so strong in these states. So, these states slip down in the league table if we use $I^{GGI}$ instead of $I^{AKS}$, except Gujarat (move up by one rank) if we use $I^{GGI}$ instead of $I^{AKS}$. It gets reversed in some of the other states due to the existence of stronger systematic cross-correlation between between-group inequalities in different dimensions. Existence of systematic cross-correlation between between-group inequalities in different dimensions existing in the multidimensional distribution can be identified by the deviation of the ranks of the states between $I^{GGI}$ and $I^{AKS}$. The ranks of almost all states taken into account according to $I^{GGI}$ instead of $I^{AKS}$ differ significantly (Spearman's rank correlation coefficient being 0.862, significant at 1% level). In this case only two states are able to retain their ranks in the league table when we use $I^{GGI}$ instead of $I^{AKS}$. This implies that local inequalities among the social groups vary significantly across states. Even after the significant change in the ranks of the states on using the index $I^{GGI}$, the positions of the poorest states do not change from the top according to the multidimensional between-group inequality.

Likewise in the urban areas, due to the existence of stronger systematic cross-correlation between inequalities in different dimensions, some states move up along the league table in case of ranking by $I^{GGI}$ instead of $I^{AKS}$. For instance, Karnataka, Tamil Nadu and West Bengal move up in the league table if we use $I^{GGI}$ instead of the relative AKS index. Some other states slip down in the league table due to weak systematic cross-correlation between between-group inequalities across dimensions. The ranking of the states differ significantly, and only five states can retain their ranks between these two measures of multidimensional inequalities. The correlation between the inter-state rankings determined by $I^{AKS}$ and $I^{GGI}$ is positive but weak (Spearman's rank correlation coefficient being 0.659, significant at 1% level). In spite of the change in the computed values of between-group inequality and ranks of the states, the positions of the poorest states Bihar, Orissa, Madhya Pradesh and Uttar Pradesh have not changed significantly according to $I^{GGI}$, i.e., these states are still occupying the top positions of the league table according to the computed values of between-group inequality in the urban areas.

**Conclusion**

In this study we have developed two multidimensional between-group inequality measures and characterized these measures on the basis of a set of axiomatic properties of multidimensional between-group inequality. After the construction of these measures of multidimensional between-group inequality we have applied these measures on Indian data and we have found significant multidimensional between-group inequality across the states of India. Another important fact has been observed that the between-group inequality in the distribution of household monthly per capita consumer expenditure and education is greater in urban areas compared to the rural areas across almost all states.

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**Appendix I**

**(A) Basic axioms in the analysis of multidimensional between-group inequality:**

(i) *Continuity* (C)
A multidimensional inequality index $I^M$ (X) should be continuous.

(ii) *Normalization* (N)
If all rows of the matrix representing the multidimensional distribution are identical, then there is no inequality in the multidimensional distribution and the value of the inequality index is zero, i.e., $I^M$ (X) = 0.

(iii) *Anonymity* (A)
As in the case of Unidimensional between-group inequality analysis, the anonymity principle has two sub-principles in the multidimensional between-group inequality analysis.

(a) *Within-group anonymity* (WIA)
A multidimensional between-group inequality measure satisfies this property if its value is invariant with the

permutations of the values of all attributes assigned to different individuals within a group. For M attributes and K groups, a measure satisfies this axiom if the interchange in the possessions of each attribute by the individuals within each group do not influence the value of the index.

(b) *Group-identity anonymity principle* (GIA)
A multidimensional between-group inequality measure satisfies this property if it is invariant after any permutation of the group identities across the distributions of all attributes.

(iv) Population replication invariance (PI):
The population replication invariance principle has three sub-principles in multidimensional between-group inequality analysis: *Population composition invariance principle* (PCI), *Total Population size invariance principle* (TPI) and *Group replication invariance principle* (GRI). These sub-principles are similar with the unidimensional between-group inequality approach (see Chapter Two).

(v) *Scale invariance* (**SI**):
A multidimensional between-group inequality measure I (X) satisfies this property, if and only if I(X) = I (CX), where C = diag ($c_1$, $c_2$, …,$c_K$) ∀ $c_i > 0$, i.e., C is a diagonal matrix. This implies that rescaling of the attributes does not affect the value of the index.

**(B) Relevant dominance/majorization criteria:**

**(i) Uniform Pigou-Dalton majorization (UPD):** For two multidimensional distributions X and Y (with K number of groups and M number of attributes), (X, Y) ∈ UPD and X (upd) Y (i.e., distribution X dominates X according to UPD if and only if X = TY, where T is a finite product of the Pigou-Dalton matrices (T = λE + (1 – λ)Q), where E is an identity matrix and Q is a permutation matrix. It is also important to state that X cannot be derived by permuting Y.

**(ii) Uniform majorization (UM):** For two multidimensional distributions X and Y, (X, Y) ∈ UM and X (um) Y if and only if X = BY, where B is a bistochastic matrix[5]. It is also important that distribution X cannot be derived by permuting distribution Y. The relation becomes strict if X cannot be derived by permuting the rows of Y. According to the examples of the four matrices given in Appendix III(A), it is easily observed that $X_1$ is UM of $Y_1$ and $X_2$ is UM of $Y_2$.

**(iii) Correlation Increasing Majorization (CIM):** For two multidimensional distributions X and Y, (X, Y) ∈ CIM and X(cim)Y if and only if Y can be derived from X by a permutation of rows and a finite sequence of correlation increasing transfers. If one of the correlation increasing transfers is strict, then Y strictly dominates X.

**(C) Correlation increasing transfer (Boland and Proschan, 1988; List, 1999):** Let there be two row vectors a = ($a_1$, $a_2$,...,$a_K$) and b = ($b_1$, $b_2$,…,$b_K$) of the matrices representing two multidimensional distributions A and B. The distribution B could be derived by a correlation increasing transfer, when for some row indices i and j (where i and j are not equal), $b_i = a_i \wedge a_j$ and $b_j = a_i \vee a_j$, and for some $s \notin \{i, j\}$ $b_s = a_s$. This correlation increasing transfer is strict if $a_i \neq b_i$ (Where $a \wedge b = (\min\{a_1, b_1\},\dots, \min\{a_K, b_K\})$ and $a \vee b = (\max\{a_1, b_1\},\dots, \max\{a_K, b_K\})$.

**(D) Local inequalities and correlation increasing/decreasing transfer:** For instance, if there are three groups in the society and three dimensions, then $X_1$, $X_2, Y_1$ and $Y_2$ are four $3 \times 3$ ordered multidimensional distributional matrices, where along the rows we measure the groups and along the columns we measure three attributes.

$$[X_1] = \begin{bmatrix} 10 & 11 & 15 \\ 6 & 8 & 7 \\ 4 & 5 & 3 \end{bmatrix} [X_2] = \begin{bmatrix} 6 & 11 & 3 \\ 4 & 8 & 15 \\ 10 & 5 & 7 \end{bmatrix}$$

$$[Y_1] = \begin{bmatrix} 8 & 9 & 10 \\ 7 & 8 & 8 \\ 5 & 7 & 6 \end{bmatrix} [Y_2] = \begin{bmatrix} 7 & 9 & 6 \\ 5 & 8 & 10 \\ 8 & 7 & 8 \end{bmatrix}$$

In the distributions $X_1$ and $Y_1$first group is the most advantaged group in either dimension and third group is the most disadvantaged group in either dimension, and the second group occupies the intermediated position in terms of either disadvantage or advantage. In the distributions $X_2$ and $Y_2$ first group is advantaged in second dimension and occupies second and third places in other dimensions. Second group occupies the first place in the distribution of third attribute, and the second and third places in the distributions of second and first attributes. Third group respectively occupies the first, second and third positions in the distributions of the first, third and second attributes. The matrices $Y_1$ are derived from $X_1$ by progressive inter-group transfers: from first to second and third groups in first dimension; first to third group in second dimension and first to second and third groups in third dimensions. Likewise, by the transfers of first attribute from third to first and second groups; second attribute from first group to third group and thirds attribute from second to first and third groups we can get the distribution $Y_2$ from $X_2$ Hence, we can compare the degrees of inequalities between the distributions $Y_1$ and $X_1$ (where inequality in the former is less than the latter), as well as between the distributions $Y_2$ and $X_2$ (where inequality in the former is less than the latter).

However, the degrees of inequalities between $X_1$ and $X_2$, or between $Y_1$ and $Y_2$ cannot be compared in a straight forward way, as the distributions $X_2$ and $Y_2$are derived by ‘correlation decreasing transfers’ from $X_1$ and $Y_1$ (This transfers reduces the systematic cross-correlations between inequalities within different dimensions). In the former two distributions the correlation between advantaged and disadvantaged positions within different dimensions is positive and strong compared to the latter distributions. In other words, in $X_1$ and $Y_1$someone who is well-off in one dimension is more likely to be well-off in other dimensions and the individuals who are badly-off in one dimension are more likely to be badly-off in other dimensions compared to $X_2$ and $Y_2$. Therefore, even if degrees of inequalities in $X_1$ and $Y_1$ are less stronger than the degrees of inequalities within $X_2$ and $Y_2$, the estimated values of inequalities in $X_1$ and $Y_1$ may be greater than the latter distributions, due to higher systematic correlation within these

[5]For any bistochastic n×n matrix, B = ($b_{ij}$), $\sum_{i=1}^{n} b_{ij} = 1$and $\sum_{j=1}^{n} b_{ij} = 1$.

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