



## RESEARCH ARTICLE

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### STATISTICAL QUANTUM MECHANICS WITH THE GROUP SU (3), WITH ANTIPERIODIC BOUNDARY CONDITIONS DEPENDING ON THE CREATION AND ANNIHILATION OPERATORS AT T≠0

**\*Dr. Salman Al- chatouri**

Associate prof.- Department of Physics - Faculty of Science - Tishreen University - Latakia – Syria

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#### ABSTRACT

In this research we introduce creation and annihilation operators in relation with the pure homogenous gauge field( global) and impulse operators. Calculate massless quarks contributions to the time evolution for the ensemble average of the square of the global operator, and of the square of the impulse operator as well as the average of the global operator. Investigate the phase transition and the critical temperature  $T_{cr}$  as well as the duration of this transition.

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#### INTRODUCTION

Treating the problems of non-equilibrium is very important [16-1]. There are currently two common methods for treating the problems of non-equilibrium of statistical quantum mechanics:

**Method 1:** This method depends on the Heisenberg representation in quantum mechanics where the operators are dependent on time. The issues of imbalance are addressed either by the dependent of Green function, or Wigner method (Semi-classical publishing).

**Method 2:** This method depends on the Schrodinger representation in quantum mechanics where the operators are not dependent on time.

The two methods are equivalent so that we write the time evolution for the ensemble average of any operator in form

$$\left\langle \hat{A}(t) \right\rangle = T_r \left( \hat{\rho} \hat{A}_H(t) \right) = T_r \left( \hat{\rho}(t) \hat{A} \right)$$

For example, in the case of the early heating of the early universe (according to a possible inflationist phase) or the description of the hadrons under limit conditions, we have studied the experimental results of a short transit phase of the quarks and gluons plasma [17-21]. QCD is the theory of strong interaction. It describes the confinement quarks and gluons at low temperature. The self-interaction of the gluons causes singular in infrared behavior. This makes the theory more complex than others.

\*Corresponding author: Dr. Salman Al- chatouri,

Associate prof.- Department of Physics - Faculty of Science - Tishreen University - Latakia – Syria

At high temperatures, quarks and gluons plasma are expected. The phase transition that begins at a critical temperature separates both phases. In this research we will develop a new numerical mathematical method to describe non-equilibrium in the gauge theory coupled to massless quarks with the group SU (3) and antiperiodic boundary conditions. The physical background was built through the process of heating the early universe (initial) and by describing the collision of heavy ions at high energies. We took the numerical mathematical method developed in [17], [22-29] and [35-47] and based on the method of the background field and the one loop approximation which transferred the study from the gauge theory coupled to massless quarks with the group SU(3) and antiperiodic boundary conditions to study a statistical quantum mechanics with the group SU(3). In turn, we introduce creation and annihilation operators in relation with the pure homogenous gauge field( global) and impulse operators. We then calculated the contribution of the quarks to the real time evolution for the ensemble average of the square of the global operator, and of the square of the impulse operator as well as the average of the global operator and we searched for the critical temperature at which the phase transition of the quarks and the gluons plasma is done.

**RESEARCH METHODOLOGY**

**"Introduction to our research in words"**

We mentioned in the introduction that we took the numerical mathematical method developed in the dissertation [17] and references [29-22] and [39-35], based on the method of the background field and one loop approximation, which transferred the study from the gauge theory coupled to massless quarks with the group SU (3) to Study Statistical Quantum Mechanics with the group SU(3) . Man considers the gauge theory coupled to massless quarks with the group SU(3) on the loop with d=3 directions, and sets the antiperiodic boundary conditions of the quark field [35]. The formulas are divided into homogeneous and non-homogeneous formulas.

The Hamilton operator is given by reference [35] as follows:

$$\hat{H}_{eff} (1) = \frac{1}{2} \left( \frac{1}{g^2(L)} + \alpha_0 + n_f f_0 \right)^{-1} \pi_i^a \pi_i^a + (\alpha_1 + n_f f_1) B_i^a B_i^a + \frac{1}{4} \left( \frac{1}{g^2(L)} + \alpha_2 + n_f f_2 \right) \left( f^{abc} B_i^b B_j^c \right)^2 + n_f f_4 S^{abcd} B_i^a B_j^b B_k^c B_m^d \tag{1}$$

Thus we transferred the study from the gauge theory coupled to massless quarks with the group SU (3) to Study Statistical Quantum Mechanics with the group SU(3).

Where  $n_f$  is the number of flavor and  $n_f = 3$

We take the numerical constants  $\alpha_0, \alpha_1, \alpha_2, f_0, f_1, f_2, f_4$  resulting from the quantization of the quark field and the gauge field :

$$\alpha_0 = 0.032715643, \alpha_1 = -0.451569918, \alpha_2 = 0.036936 \tag{2}$$

$$f_0 = -0.1357325644, f_1 = 0.0425440245, f_2 = -0.0014692028, f_4 = -0.0021133973 \tag{2'}$$

$$B_i = PA_i = \frac{1}{L^3} \int_{T^3} A_i \tag{3}$$

A is the gauge field  $B_i = PA_i = \frac{1}{L^3} \int_{T^3} A_i \tag{3}$

$$F_{ij}^a(B) = f^{abc} B_i^b B_j^c \tag{3'}$$

Where:

L is The length of the ring in all spatial directions.

$i, j = 1, 2, 3$  are indices to spatial coordinates.

$a, b, c = 1, 2, \dots, 8$  are indices to group Generators.

$S^{abcd}$  symmetric tensor is defined as follows:

$$S^{abcd} = \frac{3}{12} (d^{abe} d^{cde} + d^{ace} d^{bde} + d^{ade} d^{bce}) + \frac{2}{3} (\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) \tag{4}$$

The values of the symmetric factors  $d^{abc}$  and the values of the antisymmetric structure constants  $f^{abc}$  are given in terms of the group generators:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{4a}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

And satisfy the following relationships:

$$\left. \begin{aligned} f^{abc} &= \frac{1}{4i} T_r \left( \left[ \begin{matrix} \hat{\lambda}^a & \hat{\lambda}^b \\ \hat{\lambda}^c \end{matrix} \right]_- \right) \\ f^{abc} &= -f^{bac} = -f^{acb} = \dots \\ f^{ade} f^{bde} &= 3\delta^{ab} \\ d^{abc} &= \frac{1}{4} T_r \left( \left[ \begin{matrix} \hat{\lambda}^a & \hat{\lambda}^b \\ \hat{\lambda}^c \end{matrix} \right]_+ \right) \\ d^{abc} &= d^{bac} = d^{acb} = \dots \\ d^{ade} d^{bde} &= \frac{5}{3} \delta^{ab} \end{aligned} \right\} \tag{4c}$$

coupling The Constant is given:

$$g^{-2}(L) = -2b_0 \log(\Lambda_{ms} L) + \frac{b_1 \log[-2 \log(\Lambda_{ms} L)]}{2b_0^2} \tag{5}$$

$$b_0 = \frac{1}{(4\pi)^2} \left( \frac{11}{3} N - \frac{2}{3} n_f \right), \Lambda_{ms} = 74.1705 \text{ MeV}$$

$$b_1 = \frac{1}{(4\pi)^2} \left( \frac{14}{3} N^2 + \frac{10}{3} N n_f + (N^2 - 1) \frac{n_f}{N} \right), \quad N = 3$$

### RESULTS AND DISCUSSION

The harmonic part of the Hamiltonian operator is:

$$H_{eff(1)}^0 = \sum_{a=1}^8 \sum_{i=1}^3 \left[ \frac{1}{2} \left( \frac{1}{g^2(L)} + \alpha_0 + n_f f_0 \right)^{-1} \hat{\pi}_i^a \hat{\pi}_i^a + (\alpha_1 + n_f f_1) \hat{B}_i^a \hat{B}_i^a \right]$$

$$H_{eff(1)}^0 = \sum_{a=1}^8 \sum_{i=1}^3 \left[ \frac{1}{2} \tilde{\alpha}_0 \hat{\pi}_i^a \hat{\pi}_i^a + \frac{1}{2} \tilde{\alpha}_1 \hat{B}_i^a \hat{B}_i^a \right] \tag{6}$$

Where:

$$(7) \tilde{\alpha}_1 = 2(\alpha_1 + n_f f_1) \quad \tilde{\alpha}_0 = \left( \frac{1}{g^2(L)} + \alpha_0 + n_f f_0 \right)^{-1}$$

We define the creation and annihilation operators as follows:

$$\hat{D}_i^+ = \sqrt{\frac{\sqrt{\tilde{\alpha}_1}}{2\hbar} \frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \hat{B}_i^+ - \frac{i}{\sqrt{2\hbar} \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}} \hat{\pi}_i^+ \quad (8)$$

In natural units is  $\hbar = 1$

$$\hat{D}_i^+ = \sqrt{\frac{\sqrt{\tilde{\alpha}_1}}{2\hbar} \frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \hat{B}_i^+ + \frac{i}{\sqrt{2\hbar} \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}} \hat{\pi}_i^+ \quad (9)$$

So we have:

$$\left[ \hat{D}_i^+, \hat{D}_j^+ \right] = \delta_{ij} \delta_{ab} \quad (10)$$

$$\left[ \hat{D}_i^+, \hat{D}_j^+ \right]_- = \left[ \hat{D}_i^+, \hat{D}_j^+ \right]_+ = 0 \quad (11)$$

$$\hat{H}_{eff(1)}^0 = \hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{D}_i^+ \hat{D}_i^+ + \frac{1}{2} \right) = \hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{N}_i^+ + \frac{1}{2} \right)$$

$$\hat{H}_{eff(1)}^0 = \hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \left( \sum_{a=1}^8 \sum_{i=1}^3 \hat{N}_i^+ + 12 \right) \quad (12)$$

$$\hat{H}_{eff(1)}^0 = \hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \left( \hat{N} + 12 \right) \quad (13)$$

Where:

$$\hat{N}_i^+ = \hat{D}_i^+ \hat{D}_i^+ \quad (14)$$

$$\hat{N} = \sum_{a=1}^8 \sum_{i=1}^3 \hat{N}_i^+ \quad (15)$$

We have:

$$\begin{aligned} \hat{D}_i^+ | \dots n_i^+ \dots \rangle &= \sqrt{n_i^+} | \dots n_i^+ - 1 \dots \rangle \\ \hat{D}_i^+ | \dots n_i^+ \dots \rangle &= \sqrt{n_i^+ + 1} | \dots n_i^+ + 1 \dots \rangle \end{aligned} \quad (16)$$

$$\hat{N}_i^+ | \dots n_i^+ \dots \rangle = n_i^+ | \dots n_i^+ \dots \rangle \quad (17)$$

$$\hat{D}_i^+ | \dots 0 \dots \rangle = 0, \quad \hat{N}_i^+ | \dots 0 \dots \rangle = 0 \quad (18)$$

And shall be:



$$\begin{aligned}
 & \left( \begin{aligned}
 & \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d \\
 & + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d \\
 & + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d + \hat{D}_i^a \hat{D}_j^b \hat{D}_k^c \hat{D}_l^d
 \end{aligned} \right) \quad (22)
 \end{aligned}$$

We calculate the time evolution of the ensemble average of magnetic energy in Schrodinger's representation:

$$\begin{aligned}
 \left\langle \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{B}_i^a \hat{B}_i^a \right) (t) \right\rangle &= T_r \left( \hat{\rho}(t) \left( \sum_{a=1}^8 \sum_{i=1}^3 \hat{B}_i^a \hat{B}_i^a \right) \right) \\
 &= \sum_{a=1}^8 \sum_{i=1}^3 \left( T_r \left( \hat{\rho}(t) \hat{B}_i^a \hat{B}_i^a \right) \right) \\
 &= \sum_{a=1}^8 \sum_{i=1}^3 \frac{\hbar}{2 \sqrt{\frac{\alpha_1}{\alpha_0}}} \left( T_r \left( \hat{\rho}(t) \left( \hat{D}_i^+ + \hat{D}_i^a \right) \left( \hat{D}_i^+ + \hat{D}_i^a \right) \right) \right) \\
 \left\langle \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{B}_i^a \hat{B}_i^a \right) (t) \right\rangle &= \sum_{a=1}^8 \sum_{i=1}^3 \sum_{n_i^a} \frac{\hbar}{2 \sqrt{\frac{\alpha_1}{\alpha_0}}} \left[ (2n_i^a + 1) \rho_{n_i^a, n_i^a}(t) \right. \\
 &+ \sqrt{n_i^a} \sqrt{n_i^a - 1} \rho_{n_i^a, n_i^a - 2}(t) + \sqrt{n_i^a} \sqrt{n_i^a + 2} \rho_{n_i^a, n_i^a + 2}(t) \quad (23)
 \end{aligned}$$

We calculate the time evolution of the ensemble average of electric energy in Schrodinger's representation:

$$\begin{aligned}
 \left\langle \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) (t) \right\rangle &= T_r \left( \hat{\rho}(t) \left( \sum_{a=1}^8 \sum_{i=1}^3 \hat{\pi}_i^a \hat{\pi}_i^a \right) \right) \\
 &= \sum_{a=1}^8 \sum_{i=1}^3 \left( T_r \left( \hat{\rho}(t) \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) \right) \right) \\
 \left\langle \sum_{a=1}^8 \sum_{i=1}^3 \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) (t) \right\rangle &= \sum_{a=1}^8 \sum_{i=1}^3 \sum_{n_i^a} \frac{-\hbar \sqrt{\frac{\alpha_1}{\alpha_0}}}{2} \left[ -(2n_i^a + 1) \rho_{n_i^a, n_i^a}(t) \right. \\
 &+ \sqrt{n_i^a} \sqrt{n_i^a - 1} \rho_{n_i^a, n_i^a - 2}(t) + \sqrt{n_i^a + 1} \sqrt{n_i^a + 2} \rho_{n_i^a, n_i^a + 2}(t) \quad (24)
 \end{aligned}$$

Then we calculate the time evolution of the ensemble average of the homogeneous magnetic field operator(Global) after we introduce a start impulse on the system to become asymmetrical and then the system has been transferred twice from the equilibrium, first through the start impulse and secondly through the interactions terms then becomes the Hamiltonian operator:

$$H_{eff(1)}^{\wedge 0} = \sum_{a=1}^8 \sum_{i=1}^3 \left[ \frac{1}{2} \tilde{\alpha}_0 \left( \pi_i^a \pi_i^a - \pi^0 \right) + \frac{1}{2} \tilde{\alpha}_1 B_i^a B_i^a \right]$$

And  $\rho'$  is a symbol for the density operator depended on the new Hamilton operator

$$\begin{aligned} \left\langle \sum_{a=1}^8 \sum_{i=1}^3 B_i^a(t) \right\rangle &= Tr \left( \rho'(t) \left( \sum_{a=1}^8 \sum_{i=1}^3 B_i^a \right) \right) \\ &= \sum_{a=1}^8 \sum_{i=1}^3 \left( Tr \left( \rho'(t) B_i^a \right) \right) \\ \left\langle \sum_{a=1}^8 \sum_{i=1}^3 B_i^a(t) \right\rangle &= \sum_{a=1}^8 \sum_{i=1}^3 \sum_{n_i^a} \left( \sqrt{n_i^a + 1} \rho'_{n_i^a, n_i^a + 1}(t) + \sqrt{n_i^a} \rho'_{n_i^a, n_i^a - 1}(t) \right) \end{aligned} \tag{25}$$

Where the density matrix satisfies the equation:

$$i\hbar \frac{d\rho}{dt} = H \rho - \rho H \tag{26}$$

And therefore:

$$\begin{aligned} i\hbar \frac{d}{dt} \langle \cdot | n_i^a \cdot | \rho | \cdot | m_i^a \cdot \rangle &= \langle \cdot | n_i^a \cdot | H \rho | \cdot | m_i^a \cdot \rangle - \langle \cdot | n_i^a \cdot | \rho H | \cdot | m_i^a \cdot \rangle \\ &= \sum_{n_i^a} \left( \langle \cdot | n_i^a \cdot | H | \cdot | n_i^a \cdot \rangle \langle \cdot | n_i^a \cdot | \rho | \cdot | m_i^a \cdot \rangle - \langle \cdot | n_i^a \cdot | \rho | \cdot | n_i^a \cdot \rangle \langle \cdot | n_i^a \cdot | H | \cdot | m_i^a \cdot \rangle \right) \\ i\hbar \frac{d}{dt} \rho_{n_i^a, m_i^a} &= \sum_{n_i^a} \left( H_{n_i^a, n_i^a} \rho_{n_i^a, m_i^a} - \rho_{n_i^a, n_i^a} H_{n_i^a, m_i^a} \right) \end{aligned} \tag{27}$$

We can calculate the time evolution of the ensemble average in Equations (23),(24) and (25) numerically by the iterative solution of Equation (27).

That is why we calculate  $H_{n_i^a, m_i^a}$  from equation (22). We find:

$$\begin{aligned} LH_{n_i^a, m_i^a} &= \langle n_i^a | L\hat{H} | m_i^a \rangle = \hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \left[ \sum_{a=1}^8 \sum_{i=1}^3 m_i^a \delta_{n_i^a, m_i^a} + 12 \right] + \frac{1}{4} \left( \frac{1}{g^2(L)} + \alpha_2 + n_f f_2 \right) \\ &\sum_{a=1}^8 \cdot \sum_{b=1}^8 \cdot \sum_{c=1}^8 \cdot \sum_{i=1}^3 \cdot \sum_{j=1}^3 (f^{abc})^2 \frac{\hbar^2}{4 \left( \frac{\tilde{\alpha}_1}{\tilde{\alpha}_0} \right)} \cdot \left[ \left[ \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} \sqrt{m_i^a + 3} \sqrt{m_i^a + 4} \delta_{n_i^a, m_i^a + 4} \delta_{i,j} \delta^{ac} \delta^{ab} \right. \right. \\ &+ m_i^a \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} \delta_{n_i^a, m_i^a + 2} \delta_{i,j} \delta^{ac} \delta^{ab} + (m_i^a + 1)^{\frac{3}{2}} \sqrt{m_i^a + 2} \delta_{n_i^a, m_i^a + 2} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ m_i^a (m_i^a - 1) \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} + \sqrt{m_i^a + 1} (m_i^a + 2)^{\frac{3}{2}} \delta_{n_i^a, m_i^a + 2} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ (m_i^a)^2 \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} + (m_i^a) (m_i^a + 1) \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ \sqrt{m_i^a} \sqrt{m_i^a - 1} (m_i^a - 2) \delta_{n_i^a, m_i^a - 2} \delta_{i,j} \delta^{ac} \delta^{ab} + \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} (m_i^a + 3) \delta_{n_i^a, m_i^a + 2} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ m_i^a (m_i^a + 1) \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} + (m_i^a + 1)^2 \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ \sqrt{m_i^a} (m_i^a - 1)^{\frac{3}{2}} \delta_{n_i^a, m_i^a - 2} \delta_{i,j} \delta^{ac} \delta^{ab} + (m_i^a + 1) (m_i^a + 2) \delta_{n_i^a, m_i^a} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ \sqrt{m_i^a} \sqrt{m_i^a - 1} (m_i^a + 1) \delta_{n_i^a, m_i^a - 2} \delta_{i,j} \delta^{ac} \delta^{ab} \\ &+ (m_i^a)^{\frac{3}{2}} (m_i^a - 1) \delta_{n_i^a, m_i^a - 2} \delta_{i,j} \delta^{ac} \delta^{ab} + \left. \left. \sqrt{m_i^a} \sqrt{m_i^a - 1} \sqrt{m_i^a - 2} \sqrt{m_i^a - 3} \delta_{n_i^a, m_i^a - 4} \delta_{i,j} \delta^{ac} \delta^{ab} \right] \right] \end{aligned}$$

$$\begin{aligned}
& + n_f f_4 \sum_{a=1}^8 \cdot \sum_{b=1}^8 \cdot \sum_{c=1}^8 \cdot \sum_{d=1}^8 \cdot \sum_{i=1}^3 \cdot \sum_{j=1}^3 \cdot \sum_{k=1}^3 \cdot \sum_{m=1}^3 S^{abcd} \cdot \frac{\hbar^2}{4 \left(\frac{\alpha_1}{\alpha_0}\right)} \\
& \left[ \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} \sqrt{m_i^a + 3} \sqrt{m_i^a + 4} \delta_{n_i^a, m_i^a + 4} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \right. \\
& m_i^a \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} \delta_{n_i^a, m_i^a + 2} \delta_{i,m} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& (m_i^a + 1)^{\frac{3}{2}} \sqrt{m_i^a + 2} \delta_{n_i^a, m_i^a + 2} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + m_i^a (m_i^a - 1) \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \sqrt{m_i^a + 1} (m_i^a + 2)^{\frac{3}{2}} \delta_{n_i^a, m_i^a + 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + (m_i^a)^2 \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& (m_i^a) (m_i^a + 1) \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \sqrt{m_i^a} \sqrt{m_i^a - 1} (m_i^a - 2) \delta_{n_i^a, m_i^a - 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \sqrt{m_i^a + 1} \sqrt{m_i^a + 2} (m_i^a + 3) \delta_{n_i^a, m_i^a + 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& m_i^a (m_i^a + 1) \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + (m_i^a + 1)^2 \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \sqrt{m_i^a} (m_i^a - 1)^{\frac{3}{2}} \delta_{n_i^a, m_i^a - 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + (m_i^a + 1) (m_i^a + 2) \delta_{n_i^a, m_i^a} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \sqrt{m_i^a} \sqrt{m_i^a - 1} (m_i^a + 1) \delta_{n_i^a, m_i^a - 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& (m_i^a)^{\frac{3}{2}} (m_i^a - 1) \delta_{n_i^a, m_i^a - 2} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} + \\
& \left. \sqrt{m_i^a} \sqrt{m_i^a - 1} \sqrt{m_i^a - 2} \sqrt{m_i^a - 3} \delta_{n_i^a, m_i^a - 4} \delta_{i,m} \delta_{i,k} \delta_{i,j} \delta^{ad} \delta^{ac} \delta^{ab} \right] \quad (28)
\end{aligned}$$

Now we calculate  $\rho_{n_i^a, m_i^a}$  :

$$\left\langle n_i^a \left| \hat{\rho} \right| m_i^a \right\rangle = \int dB_i^a dB_i'^a \left\langle n_i^a \left| B_i^a \right\rangle \left\langle B_i^a \left| \hat{\rho} \right| B_i'^a \right\rangle \left\langle B_i'^a \left| m_i^a \right\rangle \right. \quad (29)$$

Since the

$$\hat{\rho} = \frac{e^{-\beta H_{eff}^0}}{T_r \left( e^{-\beta H_{eff}^0} \right)}$$

We have as [17] and [34]:

$$\begin{aligned}
\left\langle B_i^a \left| e^{-\beta H_{eff}^0} \right| B_i'^a \right\rangle &= \left[ \frac{\sqrt{\frac{\alpha_1}{\alpha_0}}}{2\pi\hbar \sinh \left( \hbar \sqrt{\alpha_1 \alpha_0} \beta \right)} \right]^{\frac{1}{2}} \exp \left\{ -\frac{\sqrt{\frac{\alpha_1}{\alpha_0}}}{4\hbar} \left[ (B_i^a + B_i'^a)^2 \tanh \left( \frac{\hbar \sqrt{\alpha_1 \alpha_0} \beta}{2} \right) \right. \right. \\
& \left. \left. + (B_i^a - B_i'^a)^2 \coth \left( \frac{\hbar \sqrt{\alpha_1 \alpha_0} \beta}{2} \right) \right] \right\} \\
T_r \left( e^{-\beta \hat{H}_{eff}^0} \right) &= Z(T, V, 8) = [Z(T, V, 1)]^8 = \left[ \frac{1}{2 \sinh \left( \frac{1}{2} \hbar \sqrt{\alpha_1 \alpha_0} \beta \right)} \right]^8
\end{aligned}$$

Thus:

$$\left\langle B_i^a \left| \hat{\rho} \right| B_i'^a \right\rangle = \frac{\left\langle B_i^a \left| e^{-\beta H_{eff}^0} \right| B_i'^a \right\rangle}{T_r e^{-\beta H_{eff}^0}}$$



$$\langle B_i^a | \hat{\rho} | B_i'^a \rangle = \left[ 2 \sinh \left( \frac{1}{2} \hbar \sqrt{\tilde{\alpha}_1 \tilde{\alpha}_0} \beta \right) \right]^8 \left[ \frac{\sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}}{2 \pi \hbar \sinh \left( \hbar \sqrt{\tilde{\alpha}_1 \tilde{\alpha}_0} \beta \right)} \right]^{\frac{1}{2}}$$

$$30 \left( \exp \left\{ -\frac{\sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}}{4 \hbar} \left[ (B_i^a + B_i'^a)^2 \tanh \left( \frac{\hbar \sqrt{\tilde{\alpha}_1 \tilde{\alpha}_0} \beta}{2} \right) + (B_i^a - B_i'^a)^2 \coth \left( \frac{\hbar \sqrt{\tilde{\alpha}_1 \tilde{\alpha}_0} \beta}{2} \right) \right] \right\} \right)$$

As both of the

$\langle n_i^a | B_i^a \rangle$  and  $\langle B_i'^a | m_i^a \rangle$  are special functions for harmonic oscillator  $H_{eff}^0$

That is:

$$\langle n_i^a | B_i^a \rangle = \Psi_{n_i^a}(B_i^a) = N_{n_i^a} H_{n_i^a} \left( \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{B_i^a}{\hbar} \right) \exp \left\{ -\frac{\sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}}{2 \hbar} (B_i^a)^2 \right\} \quad (32)$$

Where :

$$N_{n_i^a} = \left( \frac{\sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}}}{\pi \hbar} \right)^{\frac{1}{4}} \cdot \frac{1}{\sqrt{2^{n_i^a} n_i^a !}} \quad (32)$$

$$H_{2n_i^a} \left( \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{B_i^a}{\hbar} \right) = (-1)^{n_i^a} \frac{(2n_i^a)!}{n_i^a !} {}_1F_1 \left( -n_i^a ; \frac{1}{2} ; \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{(B_i^a)^2}{\hbar} \right) \quad (33)$$

$$H_{2n_i^a+1} \left( \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{B_i^a}{\hbar} \right) = (-1)^{n_i^a} \frac{2(2n_i^a+1)!}{n_i^a !} \left( \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{B_i^a}{\hbar} \right) {}_1F_1 \left( -n_i^a ; \frac{3}{2} ; \sqrt{\frac{\tilde{\alpha}_1}{\tilde{\alpha}_0}} \frac{(B_i^a)^2}{\hbar} \right) \quad (34)$$

$${}_1F_1(a; c; x) = \sum_{v=0}^{\infty} \frac{(a)_v}{(c)_v} \frac{x^v}{v!} = 1 + \frac{a}{c} \frac{x}{1!} + \frac{a(a+1)}{c(c+1)} \frac{x^2}{2!} + \dots \quad (35)$$

We replace (30) and (31) in (29) and the collection of the accounts ,we find:

$$\langle n_i^a | \hat{\rho} | m_i^a \rangle = \left[ 2 \sinh \left( \frac{\hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \beta}{2} \right) \right]^8 \left[ \frac{1}{\sinh(\hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \beta)} \right]^{\frac{1}{2}} \sqrt{\frac{1}{1 + \coth(\hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \beta)}} \left( e^{-\hbar \sqrt{\tilde{\alpha}_0 \tilde{\alpha}_1} \beta n_i^a} \right) \quad (36)$$

and is the density matrix in case  $n_i^a = m_i^a$

In case  $n_i^a \neq m_i^a$  the density matrix is zero.

## Conclusions and recommendations

1- We calculated the contribution of massless quarks to the time evolution of the ensemble average of magnetic energy

$$\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{B}_i^a \hat{B}_i^a \right) (t) \right\rangle \quad (23) \text{ and to the time evolution of the ensemble average of electrical energy } \left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) (t) \right\rangle \text{ in}$$

the relationship (24) in terms of  $\rho_{n_i^a, m_i^a}^{(t)}$ .

2. We calculated the contribution of massless quarks to the time evolution of the ensemble average of the magnetic field operator

$$\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \hat{B}_i^a (t) \right\rangle \text{ In terms of } \rho_{n_i^a, m_i^a}'^{(t)}.$$

3. The differential equation (27) can be solved numerically with the iterative solution after we calculate  $H_{n_i^a, m_i^a} \rho_{n_i^a, m_i^a}$

.And thus we get  $\rho_{n_i^a, m_i^a}^{(t)}$  Numerically this enables the numerical values to be represented  $\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{B}_i^a \hat{B}_i^a \right) (t) \right\rangle$  and

$\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) (t) \right\rangle$  Of the relations (23) and (24) Thus, a curved graph represents the contribution of massless quarks to both the

time evolution of the ensemble averages

4. It has been possible to study the change in evolution of both  $\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{B}_i^a \hat{B}_i^a \right) (t) \right\rangle$  and  $\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \left( \hat{\pi}_i^a \hat{\pi}_i^a \right) (t) \right\rangle$  at different

temperatures and deduce  $T_{cr}$  through this change. The time of the quark and gluon plasma phase can be deduced by observing evolution change for the same temperature  $T_{cr}$  after a very short time.

5. The results (3) and (4) are also valid for  $\left\langle \sum_{a=1}^3 \sum_{i=1}^3 \hat{B}_i^a (t) \right\rangle$  in relationship (25) after replacing the two results

$$\rho_{n_i^a, m_i^a}^{(t)} \text{ with } \rho_{n_i^a, m_i^a}'^{(t)} \text{ in equation (27).}$$

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