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EXPLORING CONCEPTS AND APPLICATIONS OF TAXICAB GEOMETRY

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ABSTRACT

This paper is mainly concerned with the concept of taxicab geometry and its relation to Euclidean geometry. The main objective of this paper is to specify and explore the concept and application of the taxicab geometry in relation to Euclidean geometry. It also presents some public concerns, application and their solution using taxicab geometry. Moreover, it also highlights the applications of taxicab geometry in some other fields.

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INTRODUCTION

Geometry is the mathematics of shape and space. It's about the properties of objects (their angles and surfaces, for instance) and the consequences of how these objects are positioned (where their shadows fall, how people must move around them). When adults think about geometry, they usually remember a course in the second year of high school one loaded with proofs about isosceles triangles and vertical angles. For many of us, the geometry course sounded the death knell for our progress and interest in mathematics. At its roots, geometry is not abstract rather it is fun and colorful, instructive and practical. Geometry is about real things: how big they are, whether they fit, how to find them, what they look like in a mirror. Geometry is naturally concrete. It identifies the properties of various shapes and measuring their dimensions. Oladosu (2014) states that geometry is a central aspect of the school mathematics curriculum and is crucial in the mathematics education of our children from the perspective of providing them with the opportunity to develop spatial awareness and geometric thinking, we first meet geometry through shapes and their properties. Geometry in spatial sense is vast and it takes years to develop deep understanding and encompasses many subfields.

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The word 'taxicab' was used or named by Harry Nathaniel Allen when he imported first 600 motorized taxis to be used by the New York Taxicab Company from France. He was inspired to create the name because the meter used to decide the fate of each journey is called a taximeter, which comes from the French word taximeter which in turn comes from the German taximeter. Taximeter comes from the Latin words 'taxa', means tax or charge, and metron which is Greek for measure. The taxicab geometry was introduced by Hermann Minkowski (1864-1909) over 100 years ago. Minkowski was a German mathematician and professor who studied at the Universities of Berlin and Konigsberg. He taught at several universities in Bonn, Konigsberg and Zurich. Interestingly, Albert Einstein was among his students in Zurich. He was one of the developers in "non-Euclidean" geometry, which led into Einstein's theory of relativity. Minkowski and Einstein worked together a lot on this idea. Minkowski wanted people to know that the side angle side axiom does not always hold true for all geometries (Poore, 2006). Taxicab geometry is a special kind of geometry that works on city streets. Taxicab geometry is similar to Euclidean coordinate geometry in some respects but they are different in many mathematical operations. The points, lines, angles are all the same and measured in the same way. In 1952, Karl Menger created a geometry exhibit at the Museum of Science and Industry in Chicago. For visitors to the exhibit, Menger also created a booklet entitled "You Will like Geometry" (Menger, 1952). It was the booklet in which the term "Taxicab geometry" was first used. Since its introduction in the late 1800s, taxicab geometry has undergone periods of great interest and practical application, as well as periods of marginalization. It received renewed attention in 1975 when Eugene Krause, a Mathematics professor at the University of Michigan, published a detailed book on the subject entitled “Taxicab Geometry: An Adventure in Non-Euclidean geometry. Gradually, It has remained and associated with the geometry ever since. Euclidean geometry measures distance as the crow flies. Minkowski recognized that this was not necessarily the best model for many real world situations, particularly for problems involving cities where distances are determined along blocks and not as the crow flies. Another valuable aspect of taxicab geometry is its simplicity as a non-Euclidean Geometry. It is more easily understood than much other non-Euclidean geometry. In fact, given its grid or Cartesian based orientation, it can be taught with the aid of graph paper as early as in the middle school years.In general, the taxicab distance between two points is measured as the sum of the change in horizontal and vertical directions between the two points and is calculated by using the formula:

$$D_t = |x_2 - x_1| + |y_2 - y_1|$$

Euclidean geometry which measures distance as the length of the straight line between two points, where Euclidean geometry is measured using the Pythagorean Theorem.

In Euclidean geometry the distance is thought of as “the way the crow flies". In taxicab geometry distance is thought of as the path a taxicab would take (Sexton, 2006). In Euclidean geometry the distance is measured through using Pythagorean Theorem. So, the measure of the distance between two points can be calculated by using the formula:

$$D_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MATERIALS AND METHODS

The overall objective of this paper is to explore the concept and application of the taxicab geometry in relation to Euclidean and geometry. The specific objectives of the study are to identify and differentiate between the fundamental properties of Euclidean and taxicab geometry and to address some associated problems in distance within the taxicab and Euclidean geometry. The analysis and discussion between the applications of these two taxicab and Euclidean geometric systems was made on the basis of different contemporary real life problems and from the related published documents.

RESULTS AND DISCUSSION

Some Geometrical Terms used in Taxicab and Euclidean Geometry

The taxicab geometry satisfies many of the axioms of Euclidean geometry but will not satisfy many of them including the SAS postulate.

Points will be defined as for the Euclidean plane. A point is an ordered pair of real numbers A(x, y).

Lines will be defined as for the Euclidean plane. A line is all the points that satisfy and equation of the form Ax + By = C or y = Mx + b.

Distance between two points is now measured by horizontal and vertical lines; it has some implications of other basic elements of geometry. In Euclidean, there was one shortest distance between two points. In taxicab geometry, there are many distances between two points.

Congruence of triangles in Euclidean geometry has many familiar conditions that ensure two triangles are congruent if the three axioms SAS, ASA, and AAS are satisfied. In taxicab geometry the only condition that ensures two triangles are congruent if SASAS exists.

The *angle* is the difference of direction between two straight lines. Where, the angle measure of taxicab geometry is same as Euclidean geometry.

In taxicab geometry, points and lines are equivalent to Euclidean points and lines so the incidence axioms all hold true. Similarly, the angle measure in taxicab geometry is the same as Euclidean geometry so the angle measure axioms all also hold true. The points are the same, the lines are the same, and angles are measured in the same way. Only the distance function is different. The distance between these two points A and B can be observed in the illustrated Figure-1.

Some Differences in Taxicab and Euclidean Geometry

In the taxicab geometry, the distance postulates (uniqueness, equality and zero distance) do not exists. The shortest distance between two points is not a straight line. Distance is not measured as the crow flies (diagonally), it is measure as a taxicab travels the "grid" of the city street, from block to block, vertically and horizontally, until the destination is reached. Because of this non-Euclidean method of measuring distance, some familiar geometric figures are transmitted: for example, circles become squares. Some graphical presentations of differences in Euclidean and taxicab geometry are given below:

Distance of any two Points in Euclidean and Taxicab Geometry

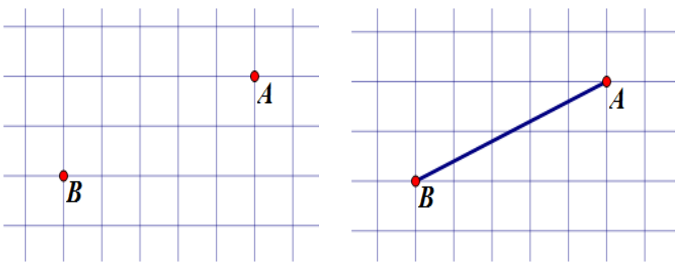

Figure 1(i)

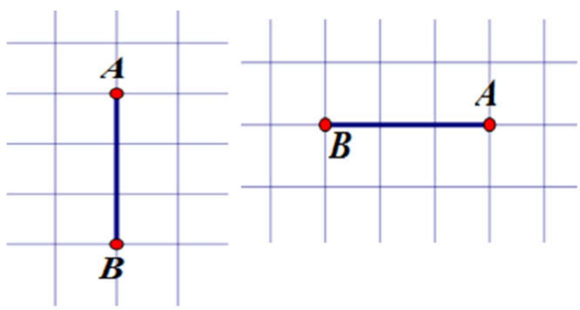

Figure 1(ii)

In the above Figure-1(i), the line joining the point A and B is the Euclidean line segment AB. According to the figure No. 1, the Euclidean space, the distance between two points A and B are obtained by using their respective value of points A and B without any restrictions.

In the above Figure-1(ii), there is no moving diagonally to reach from the point A to B or as the crow flies. The movement runs North/South (vertically) or East/West (horizontally). Movement is similar to driving on streets and avenues that are perpendicularly oriented. Taxicab geometry is the logical choice for solving the problem because people cannot walk through backyards or jump over buildings to reach in another place. In taxicab geometry the shortest distance between two points is not a straight line. The taxicab can only move vertically or horizontally in straight lines.

In taxicab geometry, the same distance can be obtained by accumulating vertical and horizontal segments as indicated in the above figure-1 like a taxi driving through the streets of a city compound of equal, adjacent square blocks. The total run is the sum of all segments, which coincides with the sum of the differences of vertical and horizontal co-ordinates of points A and B. Obviously, by observing the figure or the route of a taxi or by intuitive thinking it is easy to see that the taxicab distance is either greater or equal to the Euclidean distance of the points A and B. In a practical manner, nobody can solve such problems just as a "crow flies" (diagonally), but with the constraints that we have to stay on city streets. This means the distance formula that we are adapted to use in Euclidean geometry will not work and we use non-Euclidean geometry set up for exactly this type of problem, called taxicab geometry. This system of geometry is modeled by taxicabs travelling a city whose streets form a framework of unit square blocks.

In Euclidean geometry, distance between two points is defined as P and Q which can be shown in the figure- 3 and calculated by the formula $D_e = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

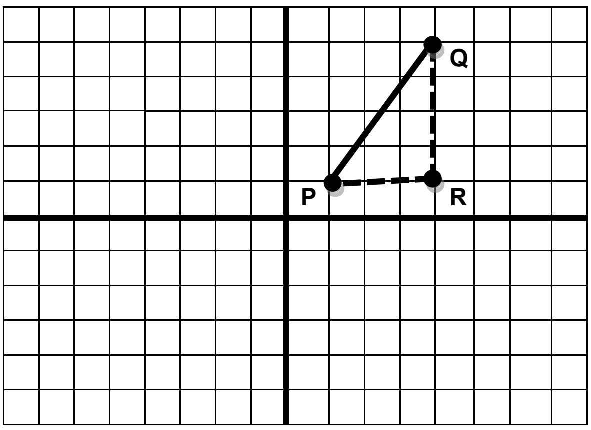

Figure 2.

The minimum distance between two points is a straight line PQ in Euclidean geometry. That is simply distance from P to Q. In taxicab geometry there may be many pathsthat join two points P and Q orto reach the given destination from P to Q. Taxicab distance between two points P and Q is the length of a shortest path from P to Q composed of line segments parallel and perpendicular to the x-axis. That is the distance from P to Q means the sum of the distance of dotted lines PR and RQ. The taxicab distance between P and Q is found by using the following formula: $D_t = PQ = |x_2 - x_1| + |y_2 - y_1|$.

In the Euclidean plane, If the coordinate of the point $P(X_1,Y_1)$ and $Q(X_2,Y_2)$ is respectively (1,1) and (4,5) thenthe Euclidean distance between the points (1,1) and (4,5) is

$$D_e = \sqrt{(x_2 - x_1)2 + (y_2 - y_1)2}$$

$$= \sqrt{(4-1)2 + (5-1)2}$$

$$= \sqrt{(3)2 + (4)2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

Similarly, the taxicab distance between the two points P and Q is

$$D_t = |x_2 - x_1| + |y_2 - y_1|$$

$$= |4\text{-}1| + |5\text{-}1|$$

$$= |3| + |4|$$

$$= 3+4$$

$$=7$$

Now we got two different distances of the same point P and Q. In taxicab geometry, the distance between P and Q is 7 that can also be found by counting the number of square blocks we would have to travel to get from P to Q from the doted path. In taxicab geometry, there are several different paths we could take to drive from point P to point Q. How many taxicab paths are possible to connect two points? We could find the answer by counting all possible paths, as shown in figure -2. Each path is represented by a square shaped box line in a 3-by-4 rectangle. Likewise, the Euclidean distance between the two points P and Q is 5 that can be found by traveling along the line connecting the points. Inherently in the calculation we use the Pythagorean Theorem to compute the distance. Actually if anybody walk on the road (taxicab distance), as shown in the above figure-2, everybody should walk 7 units length and it measure "as the crow flies" from point P to Q than it would only be 5 units length (in Euclidean distance). In taxicab and Euclidean geometry angle and side are the same, but the hypotenuse will not be the same. In general, the one and only one condition that the Euclidean and taxicab distance is equal is when both points lie along a horizontal or vertical line. If the both points P and Q lie exactly in the vertical or horizontal line then the distance will be equal in Euclidean and taxicab geometry. See Figure-3.

Congruence Property of the Triangles in Euclidean and Taxicab Geometry

The Side-Angle-Side congruence postulate for triangles i.e. if two sides and the included angle of one triangle are congruent to another triangle, then the two triangles are congruent in Euclidean geometry exists. In taxicab geometry, SAS does not work or exists i.e. the measures of the sides of a triangle are not always the actual length of the sides of the triangle. In such a case, we have a pair of triangles as shown in the Figure-4, with a side length of 2 and one right angle then it is congruent according to SAS postulate in Euclidean geometry. In the taxicab plane triangles that meet the SAS criteria are not congruent. Here, in the given Figure-4, two corresponding sides of the triangle are equal having 2 unit length but the remaining sides are not equal with one 2 unit length and the other 4 unit length. So, it is obvious that the triangles are not congruent in taxicab geometry.

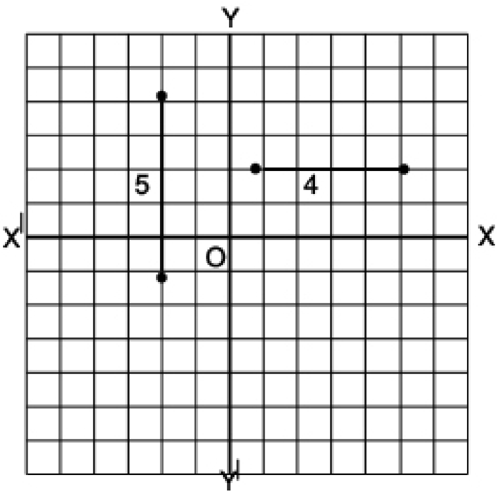

Figure 3.

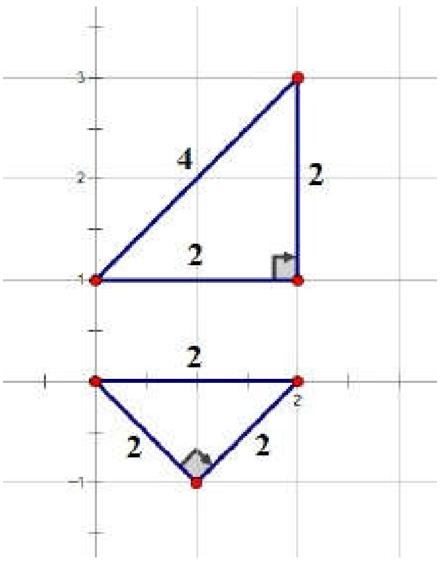

Figure 4(i) **.**

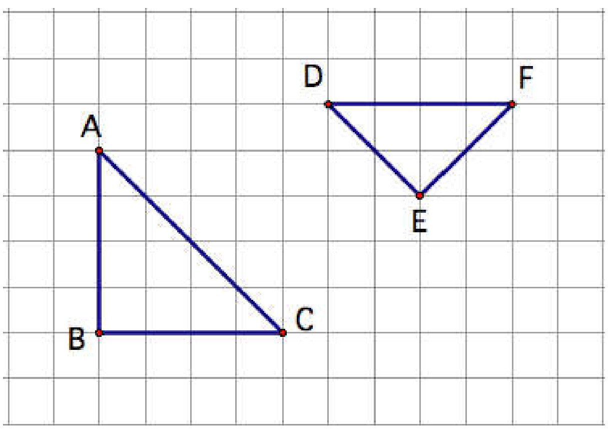

Figure 4(ii).

Applications of Taxicab Geometry

The power of mathematics denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use in a practical field effectively to solve the routine and non-routine problems in daily life. In addition, for each individual, mathematical power involves the development of personal self-confidence (Curriculum and Evaluation Standards, NCTM, 1989). Taxicab geometry has important practical applications. As Professor Krause points out, "While Euclidean geometry appears to be a good model of the 'natural' world, taxicab geometry is a better model of the artificial urban world that man has built." So, it is important for the practical applications of this non-Euclidean system to urban geometry and urban planning from deciding the optimum location for a factory/a phone booth or anything else to determining the most efficient routes for a mass transit system. The underlying emphasis throughout this unique, challenging textbook is on how mathematicians think, and how they apply an apparently theoretical system to the solution of real-world problems. The taxicab geometry can be considered as an urban planning geometry. It can address the sample problems as listed below, which is happening in day to day functioning in the modern urban area.

Problems

1. Police department receives a report of motorcycle accident at X= (-1,4). There are two police cars located in the area. Car A is at (2,1) and car B is at (-1,-1). Which car should be sent?
2. How to find the apartment for four people in a town working in four different offices stay in the same apartment in the shortest distance?
3. There are three high schools in the city. School A is at (2, 1), student B is at (-3,-3) and C is at (-6,-1). How to make the school boundaries so that each student in the City attends the school closet to them?
4. The telephone company wants to set up pay phone booths so that everyone living within 12 blocks of the center of town is within four blocks of a payphone booth. Money is tight, than how the telephone company set up the booths so that they complete in the least amounts as possible

Suppose the control room of police office receives a report of a motorcycle accident at X = (-1, 4). In police office, there are only two stands by police vans for emergency rescue for the accident in the area. Van A= (2, 1) and van B= (-1, - 1). In such situation, the police in charge should take decision fast that which van should be sent for the fastest rescue. In such condition the taxicab geometry can be the best solution. First let's see the figure, the problem to what we are looking at:

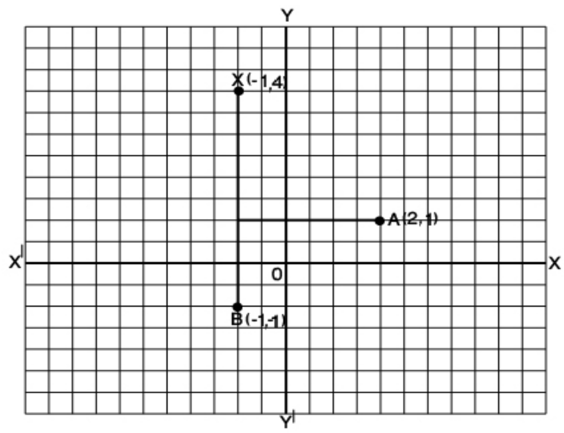

Figure 5.

The solution of the first problem given above as police vans cannot drive through peoples' houses or they can't go without road. They have to stick to the streets. Taxicab geometry will be the best choice to solve this problem. One simply needs to compare the distance in taxicab geometry from the duty in charge to each van.

The distances between motorcycle accident placesX and van A is:

D_t = [(-1,4), (2, 1)]
= |2-(-1) | + |1-4 |
= |3 | + |-3 |
=3+3
=6

The distance between motorcycle accident place X and van B is:

D_t = [(-1, 4), (-1,-1)]
= |-1-(-1) | + |-1-4 |
= |0 | + |-5 |
=0+5
=5

Thus, one can observe that, van B is one block closer to the motorcycle accident place X.

Public Concern Distance related Applications

The most interesting and important things that the change of the meaning and definition of distance used in Euclidean geometry for the last 2000 years, came possible by the use of taxicab geometry. Euclidean geometry measures distance "as the crow flies", but this rarely constitutes a good model for real-life situations in the modern time, particularly in cities, where one is only concerned with the distance their car will need to travel, or men walk and they certainly don't fly (yet). This is the most essential and inspiring application of taxicab geometry in the modern time. Taxicab distance is the right model of distance for some games played on a square grid and where only vertical and horizontal moves are allowed. (Arguably, most such games actually allow diagonal moves, in which case we have to use yet another definition of distance. Another very good reason for applying taxicab geometry is that it is a simple non-Euclidean geometry. Taxicab geometry has the advantage of being fairly intuitive compared to some other non-Euclidean geometries, and it requires less mathematical background. The concept of distance has far-reaching effects in geometry. Everything from angles to conic sections to area and volume are in some manner dependent on the meaning of distance. Having changed the distance formula for Euclidean geometry and seen some of the basic effects, one should expect the ripple effect to continue deep into other concepts. Despite all of the different subject areas of mathematics that exist, perhaps geometry has the most profound impact on our everyday lives. Everything around us has a shape, volume, surface area, location, and other physical properties. Since its origins, geometry has significantly impacted the ways people live. While we may not immediately think "geometry" when we perform everyday tasks, geometry is all around us. For instance, stop signs have the shape of an octagon, fish tanks must be carefully filled so as to prevent overflowing, and gifts need a certain amount of wrapping paper to look nice, just to name a few real-life applications. Another major application of taxicab geometry is the fine architecture and well planning of the physical world. The construction of a building and the structure of its components are important to consider in order maximizing building safety, economical and viable. It is necessary to manage the public concern offices or station like hospital, school, bus stop, railway station, telephone booth, water and sanitation champ, park police station, post office and many more should be placed as far as nearer to all the public of that locality. Taxicab geometry can be used as a model for various applications, such as optimizing driving time in cities or laying pipes, telephone cable, and many more in a home. Caballero (2006) even explains how it can be used to model the spread of forest fires and discusses how this can be used to improve computer code for these types of simulations. Thus, the application of taxicab geometry is not only to help facilitate geometrical reasoning, but can also be applicable to many individuals and their future careers.To conclude the applications of the taxicab geometry, it is easier, more convenient and practicable to measure the distance in the modern ideal city or town. This geometry only can apply in the urban setting and uses in the development of different technologies.

Conclusion

Taxicab geometry is a non-Euclidean geometry. In this geometry, the points, lines and angles are same as measured in the Euclidean geometry. Only the distance function is different. Taxicab geometry is itself useful in a number of real lives and a very useful model of urban geography. It has many applications and is relatively easy to explore. In this article, the taxicab distance and its applications in the modern society has been discussed with some examples.

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