Available online at http://www.journalijdr.com

International Journal of
DEVELOPMENT RESEARCH

IJDR

ISSN: 2230-9926

International Journal of Development Research
Vol. 4, Issue, 1, pp. 195-199, January, 2014

Full Length Research Article

A STUDY ON *d*-SYSTEM, *m*-SYSTEM AND *n*-SYSTEM IN TERNARY SEMIGROUPS

[1,*]Madhusudhana Rao, D. and [2]Sarala, Y.

[1]Department of Mathematics, V. S. R & N. V. R College, Tenali, A.P. India
[2]Department of Mathematics, KLUF University, Green Field, Vaddeswaram, Guntur (Dt), A. P. India

ARTICLE INFO

Article History:
Received 22nd October, 2013
Received in revised form
25th November, 2013
Accepted 23rd December, 2013
Published online 25th January, 2014

Key words:
Completely prime,
Prime,
Completely semiprime,
Semiprime,
d-system,
m-system,
n-system,
U-ternary semigroup.

ABSTRACT

Qualitative In this paper the terms d-system, m-system, n-system, U-ternary semigroup are introduced. It is proved that an ideal A of a ternary semigroup T is a prime ideal of T if and only if T\A is an m-system of T or empty. It is proved that an ideal A of a ternary semigroup T is completely semiprime if and only if T\A is a d-system of T or empty. It is proved that every m-system in a ternary semigroup T is an n-system. Further it is proved that an ideal Q of a ternary semigroup T is a semiprime ideal if and only if T\Q is an n-system of T (or) empty. It is proved that if N is an n-system in a ternary semigroup T and a $\in$ N, then there exist an m-system M in T such that a $\in$ M and M $\subseteq$ N. It is proved that a ternary semigroup T is U-ternary semigroup if and only if every ideal A of T is semiprime ideal of T. Further it is proved that if T is a ternary semigroup and A is an ideal of T, then T is U-ternary semigroup if and only if T\A is an n-system of T or empty and if T is U-ternary semigroup and A is an ideal of T, then T\A is an m-system of T.

INTRODUCTION

The theory of ternary algebraic systems was introduced by Lehmer (1932) in 1932, but earlier such structures was studied by Kerner (2000) who give the idea of *n*-ary algebras. Lehmer (1932) investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. Ternary semigroups are universal algebras with one associative ternary operation. LOS (Lyapin, 1981) studied some properties of ternary semigroup and proved that every ternary semigroup can be embedded in a semigroup. SIOSON (Sarala *et al*., 2013) introduced the ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroups and characterized them by using the properties of quasi-ideals. Dixit and Dewan (1995, 1997) studied quasi-ideals and bi-ideals in ternary semigroups. In this paper we characterize different types of systems in ternary semigroups.

PRELIMINARIES

DEFINITION 2.1: Let T be a non-empty set. Then T is said to be a ternary semigroup if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \rightarrow [x_1x_2x_3]$ satisfying the condition : $[(x_1x_2x_3)x_4x_5] = [x_1(x_2x_3x_4)x_5] = [x_1x_2(x_3x_4x_5)]$ $\forall$ x_i $\in$ T, $1 \le i \le 5$.

DEFINITION 2.2: Let T be ternary semigroup. A non empty subset S of T is said to be a *ternary subsemigroup* of T if $abc \in$ S for all $a,b,c \in$ S.

NOTE 2.3: A non empty subset S of a ternary semigroup T is a ternary subsemigroup if and only if SSS $\subseteq$ S.

***Corresponding author: Madhusudhana Rao, D.**
Department of Mathematics, V. S. R & N. V. R College, Tenali, A.P. India

DEFINITION 2.4: A nonempty subset A of a ternary semigroup T is said to be *left ternary ideal* or *left ideal* of T if $b, c \in$ T, $a \in$ A implies $bca \in$ A.

DEFINITION 2.5: A nonempty subset of a ternary semigroup T is said to be a *lateral ternary ideal* or simply *lateral ideal* of T if $b, c \in$ T , $a \in$ A implies $bac \in$ A.

DEFINITION 2.6: A nonempty subset A of a ternary semigroup T is a *right ternary ideal* or simply *right ideal* of T if $b, c \in$ T , $a \in$ A implies $abc \in$ A.

DEFINITION 2.7: An ideal A of a ternary semigroup T is said to be a *completely prime ideal* of T provided $x, y, z \in$ T and $xyz \in$ A implies either $x \in$ A or $y \in$ A or $z \in$ A.

DEFINITION 2.8: An ideal A of a ternary semigroup T is said to be a *completely semiprime ideal* provided $x \in$ T, $x^n \in$ A for some odd natural number n >1 implies $x \in$ A.

DEFINITION 2.9: An ideal A of a ternary semigroup T is said to be *semiprime ideal* provided X is an ideal of T and $X^n \subseteq$ A for some odd natural number n implies X $\subseteq$ A.

DEFINITION 2.1 : If A is an ideal of a ternary semigroup T , then the intersection of all prime ideals of T containing A is called *prime radical* or simply *radical* of A and it is denoted by $\sqrt{A}$ or *rad* A.

THEOREM 2.11: An ideal Q of ternary semigroup T is a semiprime ideal of T if and only if $\sqrt{Q}$ =Q.

RESULTS

DEFINITION 3.1: A nonempty subset A of a ternary semigroup T is said to be an *m*-system provided for any $a, b, c \in$ A implies that $T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \cap A \neq \emptyset$.

THEOREM 3.2: An ideal A of a ternary semigroup T is a prime ideal of T if and only if T\A is an *m*-system of T or empty.

Proof: Suppose that A is a prime ideal of a ternary semigroup T and T\A $\neq \emptyset$.
Let $a, b ,c \in$ T\A . Then $a \notin$ A, $b \notin$ A and $c \notin$ A.
Suppose if possible $T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \cap T\backslash A = \emptyset$.
$T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \cap T\backslash A = \emptyset \Rightarrow T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \subseteq A$.
Since A is prime, either $a \in$ A or $b \in$ A or $c \in$ A.
It is a contradiction. Therefore $T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \cap T\backslash A \neq \emptyset$.
Hence T\A is an *m*-system.
Conversely suppose that T\A is either an *m*-system of T or T\A = $\emptyset$.
If T\A = $\emptyset$, then T = A and hence A is a prime ideal of T.
Assume that T\A is an *m*-system of T. Let $a, b, c \in$ T and $< a > < b > < c > \subseteq$ A.
Suppose if possible $a \notin$ A, $b \notin$ A and $c \notin$ A. Then $a, b, c \in$ T\A. Sine T\A is an *m*-system,
$\Rightarrow T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \cap T\backslash A \neq \emptyset \Longrightarrow T^1T^1aT^1T^1bT^1T^1c\, T^1T^1 \nsubseteq A$
$\Longrightarrow <a><b><c> \nsubseteq$ A. It is a contradiction.
Therefore $a \in$ A or $b \in$ A or $c \in$ A. Hence A is a prime ideal of T.

DEFINITION 3.3: Let T be a ternary semigroup. A non-empty subset A of T is said to be a *d*-system of T if $a \in$ A $\Rightarrow a^n \in$ A for all odd natural number n.

THEOREM 3.4: An ideal A of a ternary semigroup T is completely semiprime if and only if T\A is a *d*-system of T or empty.

Proof: Suppose that A is a completely semiprime ideal of T and T\A $\neq \emptyset$.
Let $a \in$ T\A . Then $a \notin$ A. Suppose if possible $a^n \notin$ T\A for some odd natural number n.
Then $a^n \in$ A . Since A is a completely semiprime ideal then $a \in$ A.
It is a contradiction. Therefore $a^n \in$ T\A and hence T\A is a *d*-system .
Conversely suppose that T\A is a *d*-system of T or T\A is empty.
If T\A is empty then T = A and hence A is completely semiprime.
Assume that T\A is a *d*-system of T. Let $a \in$ T and $a^n \in$ A .
Suppose if possible $a \notin$ A. Then $a \in$ T\A.

Since T\A is a d-system, $a^n \in$ T\A. It is a contradiction. Hence $a \in$ A.
Thus A is a completely semiprime ideal of T.

DEFINITION 3.5: A non-empty subset A of a ternary semigroup T is said to be an n-system provided for any $a \in$ A implies that $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap A \neq \emptyset$.

THEOREM 3.6: Every m-system in a ternary semigroup T is an n-system.

Proof: Let A be m-system of a ternary semigroup T. Let $a \in$ A. Since A is m-system, $a \in$ A, $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap A \neq \emptyset$. Therefore *A is* an n-system of T.

THEOREM 3.7: An ideal Q of a ternary semigroup T is a semiprime ideal if and only if T\Q is an n-system of T (or) empty.

Proof: Suppose that A is a semiprime ideal of a ternary semigroup T and T\A $\neq \varnothing$.
Let $a \in$ T\A. Then $a \notin$ A.
Suppose if possible $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap T\backslash A = \emptyset$.
$T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap T\backslash A = \emptyset \Rightarrow T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq A$.
Since A is semiprime, either $a \in$ A.
It is a contradiction. Therefore $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap T\backslash A \neq \emptyset$.
Hence T\A is an n-system.
Conversely suppose that T\A is either an n-system or T\A = $\varnothing$.
If T\A = $\varnothing$ then T = A and hence A is a semiprime ideal.
Assume that T\A is an n-system of T. Let $a \in$ T and $< a > \subseteq$ A.
Let $a \in$ T\A, T\A is an n-system of T $\Rightarrow T^1T^1aT^1T^1aT^1T^1aT^1T^1 \cap T\backslash A \neq \emptyset$.
Suppose if possible $a \notin$ A. Then $a \in$ T\A. Since T\A is an m-system.
Then $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq T\backslash A \Rightarrow T^1T^1aT^1T^1aT^1T^1aT^1T^1 \nsubseteq A \Rightarrow < a > \nsubseteq A$.
It is a contradiction. Therefore $a \in$ A. Hence A is a semiprime ideal of T.

THEOREM 3.8: If N is an n-system in a ternary semigroup T and $a \in$ N, then there exist an m-system M in T such that $a \in$ M and M $\subseteq$ N.

Proof: We construct a subset M of N as follows:
Define $a_1 = a$, Since $a_1 \in$ N and N is an n-system, $(T^1T^1a_1T^1T^1a_1T^1T^1a_1T^1T^1) \cap N \neq \emptyset$.

Let $a_2 \in (T^1T^1a_1T^1T^1a_1T^1T^1a_1T^1T^1) \cap N$. Since $a_2 \in$ N and N is an n-system, $(T^1T^1a_2T^1T^1a_2T^1T^1a_2T^1T^1) \cap N \neq \varnothing$ and so on.

In general, if a_i has been defined with $a_i \in$ N, choose a_{i+1} as an element of $(T^1T^1a_2T^1T^1a_2T^1T^1a_2T^1T^1) \cap N$. Let M = $\{a_1, a_2 .....a_i, a_{i+1} ....\}$. Now $a \in$ M and M $\subseteq$ N.
We now show that M is an m-system.
Let a_i, a_j, $a_k \in$ M (for $i \leq j \leq k$).
Then $a_{k+1} \in T^1T^1a_kT^1T^1a_kT^1T^1a_kT^1T^1 \subseteq T^1T^1a_jT^1T^1a_jT^1T^1a_kT^1T^1$
$\subseteq T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1$
$\Rightarrow a_{k+1} = T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1$. But $a_{k+1} \in$ M, so $a_{k+1} \in T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1 \cap M$,
Therefore M is an m-system.

Theorem 3.9: If A is an ideal of a ternary semigroup T then $\sqrt{A}$ ={$x \in$ T: every m-system of T containing x meets A} i.e., $\sqrt{A} = \{x \in T : M(x) \cap A \neq \varnothing\}$.

Proof: Suppose that $x \in \sqrt{A}$. Let M be an m-system containing x.
Then T\M is a prime ideal of T and $x \notin$ T\M. If M $\cap$ A = $\varnothing$ then A $\subseteq$ T\M.
Since T\M is a prime ideal containing A, $\sqrt{A} \subseteq$ T\M and hence $x \in$ T\M.
It is a contradiction. Therefore M(x) $\cap$ A $\neq \varnothing$. Hence $x \in \{x \in T : M(x) \cap A \neq \varnothing\}$. Conversely suppose that $x \in \{x \in T : M(x) \cap A \neq \varnothing\}$.

Suppose if possible $x \notin \sqrt{A}$. Then there exists a prime ideal P containing A such that $x \notin$ P. Now T\P is an *m*-system and $x \in$ T\P.

A $\subseteq$ P $\Rightarrow$ T\P $\cap$ A = $\varnothing \Rightarrow x \notin \{x \in T : M(x) \cap A \neq \varnothing\}$.

It is a contradiction. Therefore $x \in \sqrt{A}$. Thus $\sqrt{A} = \{x \in T : M(x) \cap A \neq \varnothing\}$.

DEFINITION 3.10: A ternary semigroup T is said to be *U-ternary semigroup* provided for any ideal A in T, √A = T implies A = T.

EXAMPLE 3.11: Let T be the ternary semigroup under the multiplication given in the following table.

.	*a*	*b*	*c*	*d*
a	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*b*
c	*a*	*a*	*a*	*a*
d	*a*	*a*	*c*	*d*

It can be easily verified that T is U-ternary semigroup.

THEOREM 3.12: A ternary semigroup T is U-ternary semigroup if and only if every ideal A of T is semiprime ideal of T.

Proof: Suppose that T is U-ternary semigroup. Let A is a ideal of T and √A = T. √A = T and T is U-ternary semigroup implies that A = T. Therefore √A = A. By theorem 2.11, A is semiprime ideal of T.

Conversely suppose that A is semiprime ideal of ternary semigroup T and √A = T. By theorem 2.11, √A = A and hence √A = T implies that A = T. Therefore T is U-ternary semigroup.

THEOREM 3.13: If T is a ternary semigroup and A is an ideal of T, then T is U-ternary semigroup if and only if T\A is an *n*-system of T or empty.

Proof **:** Suppose that T is U-ternary semigroup. By theorem 3.12, an ideal A of ternary semigroup T is semiprime ideal of T. By theorem 3.7, T\A is an *n*-system of T or empty.

Conversely suppose that T\A is an *n*-system of T or empty. By theorem 3.7, A is semiprime ideal of T. By theorem 3.12, T is U-ternary semigroup.

COROLLARY 3.14: If T is U-ternary semigroup and A is an ideal of T, then T\A is an *m*-system of T .

Proof: Suppose that T is U-ternary semigroup. By theorem 3.13, T\A is an *n*-system of T. Therefore by theorem 3.6, T\A is an *m*-system of T.

REFERENCES

Anjaneyulu A. *Pimary ideals in semigroups* – Semigroup Form, Vol.20 (1980) 129-144.

Anjaneyulu A. *On primary semigroups – Czechoslovak Mathematical Journal*, 30(105) 1980, Praha.

Anjaneyulu A. *Semigroups in which prime ideals are maximal* – Semigroup Form, Vol.22 (1981) 151-158.

Anjaneyulu A. *Structure and ideal theory of duo semigroups* – Semigroup Form Vol.22 (1981) 257-276.

Anjaneyulu A. *Structure and ideal theory of semigroups* – Thesis, ANU (1980).

Dixit. V. N and Dewan. S., *A note on quasi and bi-ideals in ternary semigroups*, *Int. J. Math. Sci*. 18 (1995), 501 – 508.

Dixit. V. N and Dewan. S., *Minimal quasi-ideals in ternary semigroups, Indian J. Pure Appl. Math*. 28 (1997), 255 – 632.

Hewitt. E and Jzuckerman. H.S., *Ternary Operations and semigroups*-Semigroup Proc. Sympos, Wayne State Univ., Detroit, 1968, 55-83.

Iampan. A. *On ordered ideal extensions of ordered ternary semigroup*-Lobachevskii *Journal of Mathematics*, 31(1)(2010), 13-17.

Iampan. A. *Lateral ideals of ternary semigroups* – Yxpaihckkhii Mate. *Math*. Torn 4 (2007) no.4, 525-534.

Kerner. R., *Ternary Algebraic Structures and their applications*-Univ, P & M, Qurie pre print, Paris (2000), *Arixiv Math*.

Lehmer. D. H. *A ternary analogue of abelian groups, Amer. J. Math*., 59(1932), 329-338.

Lyapin. E. S. *Realization of ternary semigroups* – Russian Modern Algebra, Leningrad University, Leningrad, 1981, pp 43-48.

Lyapin. E. S. *Realization of ternary semigroups* – Russian Modern Algebra, Leningrad University, Leningrad, 1981, pp 43-48.

Mehdi Dehghanian and Mohammad Sadegh Modarres – *Ternary γ-homomorphisms and ternary* ***γ****-derivations on ternary semigroups – Journal of Inequalities and applications* 2012: 34.

Mohammad Shabir and Mehar Bano- *Prime Bi-ideals in Ternary semigroups*, *Quasi groups and Related systems* (16) (2008) 239-256.

Sarala. Y, Anjaneyulu. A and MADHUSUDHANA RAO.D. *Ternary semigroups*, - *International Journal of Mathematical Sciences, Technology and Humanities* 76 (2013) 848-859.

Sarala. Y, Anjaneyulu. A and MADHUSUDHANA RAO.D., *ideals in ternary semigroups* - *International eJournal of Mathematics and Engineering* 203 (2013) 1950-1968.

Sarala. Y, Anjaneyulu. A and MADHUSUDHANA RAO. D. *Prime radicals in ternary semigroups* - *International organization of scientific research journal of Mathematics* ISSN: 2278-5728. Volume 4, Issue 5 (Jan.-Feb. 2013), pp 43-53.

Sarala. Y, Anjaneyulu. A and Madhusudhana Rao. D. *pseudo symmetric ideals in ternary semigroups* - *International Referred Journal of Engineering and Science* ISSN (Online) 2319-183X, (print) 2319-1821 Volume 1, Issue 4 (December 2012), pp. 33-43.

Sarala. Y, Anjaneyulu. A and Madhusudhana RAO. D. *Pseudo Integral ternary semigroups –Asian Journal of Current Engineering and Maths* 2:1 Jan-Feb (2013) 40-43.

Sioson. F. M. *Ideal theory in ternary semigroups,* Math. Japon.10 (1965), 63-84.

****