



**Full Length Research Article**

**BACKBENDING AND SHAPE CHANGES IN ODD-A LANTHANUM ISOTOPES (LA)**

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**ABSTRACT**

We have developed a special computing code for calculation of nuclear shape changes and quadruple moments (Q) of Lanthanum Isotopes. It has been shown from these calculations that by increasing neutron number, deformation parameter also increase for heavier isotopes which means more deformation from spherical shape. By comparison with Nilsson level diagrams we can infer quadruple deformation parameter ( $\beta_2$ ) and calculate quadruple moments of these isotopes.

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**INTRODUCTION**

We know that nuclei in many cases have large quadruple moments (Q) and they don't behave like a point charge, rather a spherical or elliptical shape with an axis of symmetry is considered for these nuclei. By knowing the quadruple moments, we can measure deformation parameters which can be used to define the shape of nuclei. There are different theoretical and experimental methods for calculation and measurement of nuclear electric quadruple moments (DeShalet and Feshbach, 1990; Mårtensson-Pendrill, 1992; Otsuka *et al.*, 1993; Baumann *et al.*, 1998; Brown and Wildenthal, 1987; Cederberg *et al.*, 1998; Nörtershäuser *et al.*, 1998). In this paper we present a new method for calculation of quadruple moments of odd-A Lanthanum isotopes. By study of rotational gamma-decay cascades in different bands of these isotopes and drawing the experimental yrast level energies versus moments of inertia for each band, we look for back bending phenomenon (Krane, 1988) for each La isotope. If there is a back bending, then it means that there is a change of moment of inertia, which is happening by excitation of a nucleon to another state with different angular momentum, thus changing the total spin of nucleus. By comparison with related Nilsson diagram (Nilsson, 1955), we can find the location of

displaced nucleon and thus find the related quadruple deformation parameter ( $\beta_2$ ) at that excitation energy. By finding the deformation parameter we can calculate the quadruple moment of the deformed isotope and study shape changes.

**Theoretical Calculation and Discussion**

Nuclei can be obtained in very high angular momentum states, mainly through heavy-ion induced reactions (HI, xn). The states that are populated subsequently decay, through a series of statistical low-spin transitions, into the high-spin lower energies yeast structure. It has been shown that a large amount of angular momentum can be obtained by collective motion (i.e. a coherent contribution of many nucleons to the rotational motion). It is important that the nucleus exhibits a stable, deformed shape. Subsequently, rigid rotation will contribute angular momentum J and energy E according to the expression

$$E = \frac{\hbar^2}{2I} J(J + 1) \quad (1)$$

where I is the moment of inertia.

Besides the collective rotational motion, angular momentum can be acquired by non-collective motion. Here, the alignment of the individual nuclear orbits along the nuclear symmetry

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axis contributes to the total nuclear spin. The system does not have large deformed shapes but remains basically spherical or weakly deformed. The excited states should cascade down toward the ground state through a sequence of E2 gamma transitions. The observation of these cascade E2 transitions provides a way to study these excited states. In particular, we can study whether the assumption of a fixed constant moment of inertia remains valid at such high excitations. One way to test this assumption is to plot the energies of the states against  $I(I + 1)$  and to see if the slope remains constant. Figure 1 is an example of such a plot for  $^{158}\text{Er}$  and  $^{174}\text{Hf}$  nuclei and as it can be seen there appears to be some deviation from the expected linear behavior (Krane, 1988).

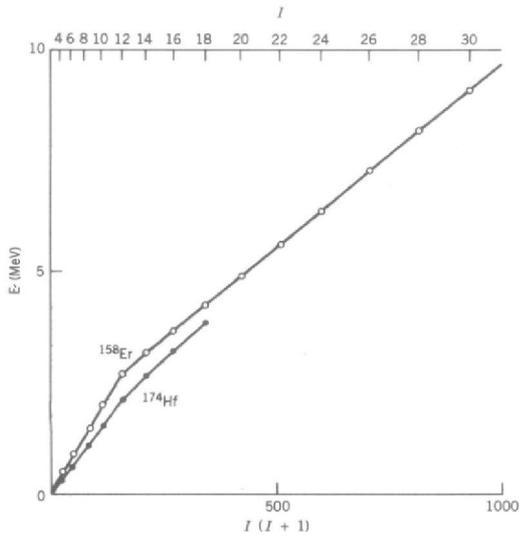


Fig. 1. E versus  $I(I + 1)$  plot for Er and Hf nuclei (Krane, 1988).

If we assume that the moment of inertia is not constant but increases gradually as we go to more rapidly rotating states. This effect known classically as "centrifugal stretching" would not occur for a rigid rotor but would occur for a fluid. Because rotating nuclei have moments of inertia somewhere between that of a rigid rotor and of a fluid, it is not surprising that centrifugal stretching occurs. There is a more instructive way to plot the data on the rotational structure. From equation (1) the energy of a transition from state  $I$  to the next lower state  $I - 2$  is

$$E(I) - E(I - 2) = \frac{\hbar^2}{2\mathcal{I}}(4I - 2) \tag{2}$$

Or by rearranging the terms

$$\frac{2\mathcal{I}}{\hbar^2} = \frac{4I - 2}{E(I) - E(I - 2)} \tag{3}$$

By plotting the left hand side of the above equation versus the square of rotational frequency  $\omega^2$ , there appears to be a gradual increase in moment of inertia among the lower angular momentum states, then a radical change in behavior and then again a return to the gradual stretching as shown in Figure 2. This effect which is known as back bending occurs in some heavy nuclei because the rotational energy exceeds the energy needed to break a pair of coupled nucleons. When this effect occurs, the unpaired nucleons go into different orbits and change the nuclear moment of inertia (Krane, 1988).

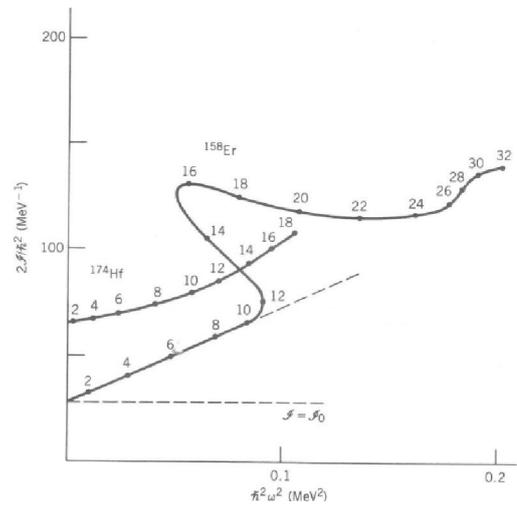


Fig. 2. Moment of inertia versus  $\omega^2$  showing back bending (Krane, 1988)

### Shape Changes in Lanthanum Isotopes

Lanthanum Isotopes have 57 protons. In this paper we studied rotational gamma decays of Isotopes from  $A=127$  to  $A=147$ . Figure 3 clearly shows Back bending for  $A=127$  Isotope. Using this plot and comparison with Nilsson diagram for neutrons with  $N = 70$ , we find quadruple deformation parameter for this isotope  $\beta_2 = -0.157$ . From this finding and using the Grodzins formula for quadruple moment (Grodzins, 1962)

$$Q \cong \frac{3}{\sqrt{5\pi}} Zr_0^2 A^{\frac{2}{3}} \beta_2(4)$$

We can calculate the quadruple moment, which is a measure of deformation of nucleus from spherical shape. The changes in deformations and calculated quadruple moments for odd-A Lanthanum isotopes are summarized in Table 1.

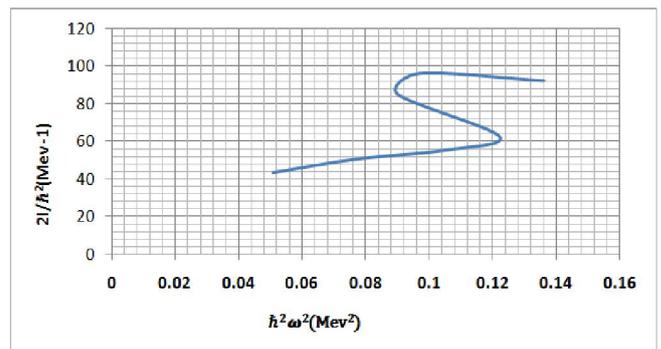


Fig. 3. Back bending for  $A=127$  Lanthanum isotope

Table 1. Deformation parameters and Quadruple moments for Lanthanum isotopes

Mass Number (A)	Deformation Parameter ( $\beta_2$ )	Quadruple Moment (Q)
127	-0.157	-246.45
129	-0.115	-174.52
131	0.147	224.39
133	0.315	510.05
135	0.378	588.80
137	0.336	528.47
139	0.450	673.35
141	0.290	471.37
143	0.340	560.70
145	0.389	651.50
147	0.420	692.30

As it can be seen from the above table, with increasing neutron numbers in Lanthanum isotopes, changes in shape occur from oblate (negative Q.M) to prolate (positive Q.M). With the exception of A= 137 and 141, there is an increasing in quadruple moments. These exceptions may be due to shell closure near N=82.

### Conclusions

It has been shown from these calculations that by increasing neutron number of Lanthanum isotopes, deformation parameter also increase for more heavier isotopes which means more deformation from spherical shape. By comparison with Nilsson level diagrams we can infer deformation parameter ( $\beta$ ) and calculate quadruple moments of these isotopes. This means that there are shape changes from oblate to prolate deformations in these isotopes. With the exception of A= 137 and 141, there is an increasing in quadruple moments. These exceptions may be due to shell closure near N=82.

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