

EVEN ODD AND DIFFERENCE PROPERTY(S) OF FIBONACCI NUMBERS

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ABSTRACT

A In mathematics, the Fibonacci numbers are the numbers in the following integer sequence, called the Fibonacci sequence, and characterized by the fact that every number after the first two is the sum of the two preceding ones:[1][2]
1,1,2,3,5,8,13,21,34(1)
Often, especially in modern usage, the sequence is extended by one more initial term:
0,1,1,2,3,5,8,13,21,34,.....[3] (2)
From the plain mathematical observation, the Fibonacci series has even-odd and difference property. The even-odd property means that the series first number is always even and further two numbers are always odd when we consider (2).But if we consider (1) the then first two numbers are always odd and next number is always even. Also, the difference property means that by considering (2), when we subtract (n+1) number with nth number the preceding number comes. The astounding fact is that it generates the total Fibonacci series again. Also, I have implemented the above properties and generated the Fibonacci series using c programming language.

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INTRODUCTION

The sequence F_n of Fibonacci numbers is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2},$$

with seed values^{[1][2]}

$$F_1 = 1, F_2 = 1$$

or^[5]

$$F_0 = 0, F_1 = 1$$

The Fibonacci sequence is named after Italian mathematician Leonardo of Pisa, known as Fibonacci. His 1202 book *Liber Abaci* introduced the sequence to Western European mathematics,^[6] although the sequence had been described earlier in Indian mathematics.^{[7][8][9]} The sequence described in *Liber Abaci* began with $F_1 = 1$. Fibonacci numbers are closely related to Lucas numbers L_n in that they form a complementary pair of Lucas sequences $U_n(1, -1) = F_n$ and $V_n(1, -1) = L_n$. They are intimately connected with the golden ratio; for example, the closest rational approximations to the ratio are $2/1, 3/2, 5/3, 8/5, \dots$.

Fibonacci numbers appear unexpectedly often in mathematics, so much so that there is an entire journal dedicated to their study, the *Fibonacci Quarterly*.

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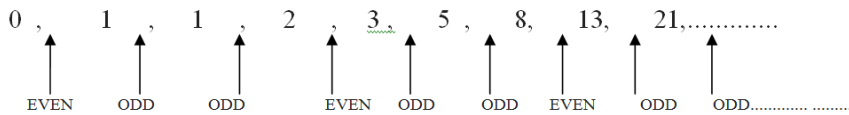
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Applications of Fibonacci numbers include computer algorithms such as the Fibonacci search technique and the Fibonacci heap data structure, and graphs called Fibonacci cubes used for interconnecting parallel and distributed systems. They also appear in biological settings,^[10] such as branching in trees, phyllo taxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple,^[11] the flowering of an artichoke, an uncurling fern and the arrangement of a pine cone's bracts.

EVEN- ODD PROPERTY

When we look at Fibonacci series it is evident that the first member of series is even when considering (2) and odd when considering (1). So, there is this same pattern in the whole Fibonacci series which is of profound importance and significance in understanding the inner beauty of Fibonacci series as it is widely used by nature.

when considering series(2), we have



Here we see clearly that Fibonacci series has symmetric even-odd property. If first digit is even other two digits or numbers are definitely odd.

Equal Summation Property

The sum property states that the addition of the first ten numbers (n=10) of Fibonacci digits that are even will be equal to the sum of odd digits in the this range. For example, if we take first ten Fibonacci series numbers,

i.e; 0, 1, 1, 2, 3, 5, 8, 13, 21, 34

Here, Even Numbers=0,2,8,34

Total even numbers=4

Sum of even numbers=44(3)

Now, Odd numbers=1,1,3,5,13,21

Total odd numbers=6

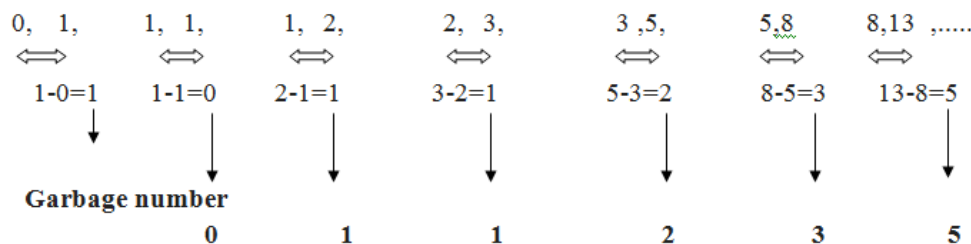
Sum of even numbers=44(4)

Theorem 1.1: Given a Fibonacci Series with n=10(first ten series numbers) such that n belongs to the set of whole numbers, the sum of even numbers in the series is always equal to the sum of odd numbers in this range.

Difference Property

The difference property states that the subtraction between next term and previous term in series always generates a Fibonacci series again (ignoring first number, this number is calles as garbage number). For example;

Given Fibonacci Series,



This is the Fibonacci series, isn't it fantastic result.....

Theorem: 1.2: Given a Fibonacci series that starts or having an initial digit 0 ,the difference property generates Fibonacci series having initial digit 1 and vice-versa.

Implementation

Here, the implementation is done using c programming language. The implementation step consists of four modules i.e;

Module 1: Generating Fibonacci series
 Module 2: Implementing even-odd property
 Module 3: implementing difference property
 Module 4: Implementing equal summation property(for n=10)

These are as under:

Module 1: Generating Fibonacci series

```

1 /*Fibonacci Series*/
2 /*0,1,1,2,3,5,8,13,21,34,55,89,144,233...*/
3 /*DATED: 17-11-2017*/
4 #include<stdio.h>
5 #include<conio.h>
6 int main()
7 {
8
9     int i, n, t1 = 0, t2 = 1,add,sum1=0,sum2=0;
10
11
12     printf("Enter the number of terms:");
13     scanf("%d",&n);
14     printf("Fibonacci Series:\n ");
15     for (i = 1; i <= n; ++i)
16     {
17         printf("%3d\n", t1);
18         add = t1 + t2;
19         t1 = t2;
20         t2 = add;
21     }
22     getch();
23     return 0;
24 }
25

```

Module 2: Implementing even-odd Property

```

1 /* EVEN-ODD PROPERTY OF FIBONACCI SERIES*/
2 /*0 (EVEN) ,1 (ODD) ,1 (ODD) ,2 (EVEN) ,3 (ODD) ,5 (ODD) ...*/
3 /*DATED: 17-11-2017*/
4 #include<stdio.h>
5 #include<conio.h>
6 int main()
7 {
8     int i, n, t1 = 0, t2 = 1,add,sum1=0,sum2=0;
9     printf("Enter the number of terms:");
10     scanf("%d",&n);
11     printf("Fibonacci Series:\n ");
12     for (i = 1; i <= n; ++i)
13     {
14         printf("%3d\n", t1);
15         if(t1%2==0)
16         {
17
18             printf("even=%d\n",t1);
19         }
20     else
21     {
22         printf("odd=%d\n",t1);
23     }
24         add = t1 + t2;
25         t1 = t2;
26         t2 = add;
27     }
28     getch();
29     return 0;
30 }

```

Module 3. Implementing Difference Property

```

1  /*DIFFERENCE PROPERTY OF FIBONACCI NUMBERS*/
2  /*DATED: 17-11-2017*/
3  #include<stdio.h>
4  #include<conio.h>
5  int main()
6  {
7
8  int i, n, t1 = 0, t2 = 1, add, sub;
9
10
11     printf("Enter the number of terms:");
12     scanf("%d", &n);
13     printf("Fibonacci Series:\n ");
14     for (i = 1; i <= n; ++i)
15     {
16         printf("%3d\n", t1);
17         sub=t2-t1;
18         printf("sub=%5d\n", sub);
19         add = t1 + t2;
20         t1 = t2;
21         t2 = add;
22     }
23     getch();
24     return 0;
25 }
26

```

Module 4. Implementing equal summation property(for n=10)

```

1  /* Summation Equality Property of Fibonacci Series*,
2  /*0,1,1,2,3,5,8,13,21,34,55,89,144,233...*/
3  #include<stdio.h>
4  #include<conio.h>
5  int main()
6  {
7
8     int i, n, t1 = 0, t2 = 1, add, sum1=0, sum2=0;
9     printf("Enter the number of terms:");
10    scanf("%d", &n);
11    printf("Fibonacci Series:\n ");
12    for (i = 1; i < 11; ++i)
13    {
14        printf("%3d\n", t1);
15        if(t1%2==0)
16        {
17
18            printf("even=%d\n", t1);
19            sum1=sum1+t1;
20            printf("sum1=%d", sum1);
21        } else{
22            printf("odd=%d\n", t1);
23            sum2=sum2+t1;
24            printf("sum2=%d", sum2);
25        }
26        add = t1 + t2;
27        t1 = t2;
28        t2 = add;
29    }
30    getch();
31    return 0;

```

RESULTS

Output of Module 1

```
C:\Users\welcome\Desktop\C PROGRAMS\Ppresentations\FSSD.exe - - - X
Enter the number of terms:12
Fibonacci Series:
0
1
1
2
3
5
8
13
21
34
55
89
```

Output of Module 2

```
C:\Users\welcome\Desktop\C PROGRAMS\Ppresentations\FSEO.exe - - - X
Enter the number of terms:10
Fibonacci Series:
0
even=0
1
odd=1
1
odd=1
2
even=2
3
odd=3
5
odd=5
8
even=8
13
odd=13
21
odd=21
34
even=34
```

Output of Module 3

```
C:\Users\welcome\Desktop\C PROGRAMS\Ppresentations\FS11.exe - - - X
Enter the number of terms:10
Fibonacci Series:
0
sub= 1
1
sub= 0
1
sub= 1
2
sub= 1
3
sub= 2
5
sub= 3
8
sub= 5
13
sub= 8
21
sub= 13
34
sub= 21
```

OUTPUT OF MODULE 4

```

C:\Users\welcome\Desktop\C PROGRAMS\Ppresentations\fs1.exe
Enter the number of terms:10
Fibonacci Series:
0
even= 0
sum1= 0 1
odd= 1
sum2= 1 1
odd= 1
sum2= 2 2
even= 2
sum1= 2 3
odd= 3
sum2= 5 5
odd= 5
sum2= 10 8
even= 8
sum1= 10 13
odd= 13
sum2= 23 21
odd= 21
sum2= 44 34
even= 34
sum1= 44_
  
```

Here, it is clearly shown that sum1 is equal to sum 2 i.e. 44 for n=10.

CONCLUSION

This work is preliminary in nature and it really focuses on finding interesting patterns in Fibonacci series. But what I found so amazing is that every famous number series has some hidden patterns and those patterns are scattered in nature providing their services. So, there are lot of further research areas such as application of Fibonacci series in machine learning, computational mathematics, computational biology, economics, etc.

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