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STATIONARITY OF ELECTROMECHANICAL PROPELLERS VARIABLES: A UNIT ROOT TEST APPROACH

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ABSTRACT

Valid mathematical models can be obtained by the means of the system's input and output data, as well as by using system identification techniques. However, it is fundamental that the data within the time series does not violate the statistical assumptions of the series. It is necessary to perform tests to detect possible violations of the system's variables. Concerning the time series of the experimental data, the present work investigates and performs three stationarity verifications tests on the data series: the ADF test (Augmented Dickey and Fuller); the PP test (Phillips and Perron); and the KPSS test (Kwiatkowski Philips Schmidt and Shin). Electrical current and angular speed data of an electromechanical propeller were collected for the testing by the means of an experimental platform. The propellers utilized are those of Unmanned Aerial Vehicles (UAVs). This work's contribution consists in establishing pathways that allow obtaining non-spurious mathematical models that represent the physical model with reliability.

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INTRODUCTION

System identification proposes to obtain and build mathematical models that explain the cause and effect relationship among the sampled data of the input and output variables. According to Aguirre (2007): "System identification the knowledge area that studies and develops techniques and algorithms to obtain models for dynamic systems from data generated by the system itself". Data can be defined as measurements or observations that characterize variables expressed by time series with a constant sampling rate. The data is acquired from real or experimental systems, or through computer program simulations.

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These variables, or the time series, intrinsically contain the modifications of the material properties, geometric dimensions and other unexplainable phenomena of the system. Their modification reflects changes on the mathematical model performance. Therefore, this analysis and mathematical modelling are important in many knowledge areas like econometrics, engineering, natural sciences and others. In order to obtain the simple static regression mathematical models, like the ARIMA (Autoregressive Integrated Moving Average) family, it is necessary that the data within the time series does not violate the statistical assumptions of the series. Because, if that happens, the mathematical model result is compromised (Margarido and Junior, 2006). Thus, it becomes indispensable to perform tests that detect possible violations on the data from the system´s variables. In this context, the present work investigates and performs tests to verify the stationarity of the data series.

The studied tests were: the ADF test (Augmented Dickey and Fuller, 1979); the PP test (Phillips and Perron, 1988); and the KPSS test (Kwiatkowski Philips Schmidt and Shin, 1992). The tests were applied on the data series of the variables electric current and angular speed, of electromechanical propellers, both relevant for the unmanned aerial vehicles (UAVs) studies. In this way, this work´s objective is to investigate the applicability of the mentioned tests, and on a second moment, to elaborate consistent and non-spurious mathematical models. The work is organized in the following way: On the first session, the materials and methods are described, as well as a brief theoretical grounding, to later on present the methodology used on the research. On the sequence, the results are presented with their respective discussion and, at the end, the conclusion is stated.

Theoretical Grounding

A time series is a set of observations acquired sequentially through time or in another physical quantity. If the dataset is continuous, the time series will be continuous. In case the data is discrete, the time series will be discrete. The time series can also be deterministic, if it´s possible to represent it by a mathematical function. It can also be classified as a stochastic process. This process is characterized as a statistical phenomenon involving probability laws (BOX, JENKINS and REINSEL, 1994). The time series techniques are based on identifying patterns on the data that can be used in future value calculations further on. Mathematical modelling can describe a time series by the means of mathematical equations. However, when a time series regression is performed, a problem can occur which is called spurious regression, that is, a "nonsense" regression (GUJARATI, 2006). That is directly connected with the stationarity of the studied series. According to Santos (2007), stationarity can be obtained by a process where all the relevant parameters for the dynamics of the system are fixed and constant throughout the observation period. Agreeing with that, Diniz (1998) states that a time series is stationary if the random data oscillate around a constant value. That is verified when the probabilistic distribution parameters like the average E(Yt) = μ, which indicates the average value of the data; the variance var (Yt) = σ^2, which represents the dispersion level of the data in relation with the average value; and the covarianceγ t, which relates the dispersion level between a data value and its subsequent; are fixed and constant throughout time (GUJARATI, 2006). Therefore, to evaluate stationarity, it is necessary to verify the existence of roots in the delay operators inside the unit circle. The ADF (Augmented Dickey and Fuller) is a null hypothesis statistical test. When the series has a unit root, it is said non-stationary; in case it does not have a unit root, it is said stationary. This test is based on the regression of the model defined by equation (1).

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta Y_{t-i} + \varepsilon_t, \quad (1)$$

β_1 is defined as the independent term (intercept or displacement); β_2 as the trend coefficient; δ as the unit root presence coefficient; ε_t as the white noise term and the $\alpha_i$as the coefficients of ΔY_{t-1} used to approximate the ARMA (Autoregressive Moving Average) structure to the errors. Still, $\Delta Y_{t-1} = Y_{t-1} - Y_{t-2}$, $\Delta Y_{t-2} = Y_{t-2} - Y_{t-3}$are defined, and so on. It is a model with constant and trend, and with its own shifted and differentiated variable, ensuring that the residue do not show autocorrelation. The number of lags, p, utilized on the series is obtained by the Schwert mathematical expression (1989) defined in equation (2), where N is the number of data values on the series.

$$p_{max} = \left[12 \left(\frac{N}{100} \right)^{\frac{1}{4}} \right] \quad (2)$$

In order to test the null hypothesis, equation (1) is estimated using least squares and the τ statistic (Dickey-Fuller, 1979) is examined. If the value of the statistic, intrinsically calculated on the ADF test, is greater than the absolute value charted by Dickey-Fuller, the null hypothesis is accepted and the series is non-stationary. The PP test (Phillips and Perron), is defined as:

"...a non-parametric procedure related to nuisance parameters, which are present in several classes of time series that have a unit root. It includes heterogeneous ARIMA (Autoregressive Integrated Moving Average) models, as well as identically distributed innovations. This method has, apparently, significant advantages when there are moving average components on the time series, and related to this, at least, offers a promising alternative to the Dickey Fuller and Said Dickey procedures" (PHILLIPS and PERRON ,1987).

The PP test is described by the same equation of the ADF test (Equation 1). In this case, the Z statistic is calculated by equation (3):

$$Z = n\hat{\delta}_n - \frac{n^2\sigma^2}{2s_n^2}(\hat{\gamma}_n^2 - \hat{\gamma}_{0,n}) \quad (3)$$

It is noted that Z is an adjustment on the Dickley and Fuller statistic. In case the process is not correlated, the covariances are null, and on that case, $\hat{\gamma}_n^2 = \hat{\gamma}_{0,n}$. If the process is not heteroscedastic, $se(\delta) = \frac{1}{n}$ so Z is obtained by equation (4):

$$Z = n\hat{\delta}_n = \frac{\hat{\delta}}{se(\hat{\delta})} \quad (4)$$

that means, Z is the Dickley and Fuller statistic and therefore, has the same statistical distribution of the ADF test. Holden and Perman (1994) emphasize that, when the noise term has moving average positive components, the ADF test power is small, if compared to the Phillips and Perron test, so that it's better to use the PP test. On the other hand, when there are negative signed moving average components, there's an indication that the Z statistic presents distortions in the case of finite sized samples. The KPSS (*Kwiatkowski Philips Schmidt* and *Shin*) test was created by Denis Kwiatkowski, Peter C. B. Phillips, Peter Schmidt and Yongcheol Shin (KWIATKOWSKI, D. et al, 1992). The test aims at determining the stationarity of a time series. The test hypothesis are:

H_0 = "The series is stationary"
H_1 = "The series presents unit root".

It can be noticed that the hypothesis of this test are not the same as those of the ADF and PP, for stationarity. On this test (KPSS), the null hypothesis is that the series is stationary. Equations (5) e (6) express its most simple version:

$$y_t = \xi D_t + r_t + \varepsilon_t \quad (5)$$

$$r_t = r_{t-1} + \mu_t \quad (6)$$

Variable r_t is a random ride, its initial value r_0 is fixed and serves as an intercept. μ_t, is a Normal and Identically Distributed Distribution (0, σ^2) (Kwiatkowski et. al, 1992). The asymptotic distribution of the statistic is derived under the null and alternative hypothesis with general conditions about the stationary error, and the hypothesis test is based on the LM statistic (Kwiatkowski et. al, 1992; Wang, 2006) according to equation (7).

$$LM = \left(\frac{1}{N^2}\right)\left(\sum_{t=1}^{N}\frac{S_t^2}{\sigma_k^2}\right) \quad (7)$$

where$S_t = \sum_{i=1}^{N} e_i$, for $t = 1,2,\dots,N$. It is considered $e_t, t = 1,2,\dots,N$, the $Y_t$regression residues as intercept, and σ_k^2as the estimated error of the regression's variance, given by equation (8).

$$\sigma^2(p) = \frac{1}{N}\sum_{t=1}^{N} {e_t}^2 + \frac{2}{N}\sum_{j=1}^{p} w_j(p) \sum_{t=j+1}^{N} e_t e_{t-j} \quad (8)$$

(Regional University of the Northwest of Rio Grande doSul). It consists of a metallic support base, with a fixed arm that operates as the support for the electromechanical propulsion system. Figures 1(a) e 1(b) show the pictures of the platform and the brushless motor under test, instrumented with the optical sensor for motor speed acquisition. The experimental platform works by the swinging principle; the motor and a load cell are positioned at the extremity of the arm. Table 1 presents the physical characteristics of the platform components**.** The connection between the experimental platform, the computer and the user is achieved by the Arduino Uno.

This device is completely electronic and its function is to acquire and verify data like rotational speed (ω), current drawn by the motor (i) and duty cycle *(D)*of the PWM (Pulse Width Modulation), which controls the motor speed. The rotational speed signal and the square wave generation are captured by the optical sensor presented on Figure 1. The square wave is sent to a converter, which converts it into an analog signal that varies linearly according to the wave frequency. The conversion into RPM is made by the reading of the analog signal. Figures 2(a) and 2(b) show, respectively, the behavior of the data obtained for the current *i(k)*and for the angular speed*ω(k)*. Electronic noise can be seen in the data of both graphs. The average value and variance were calculated after data collection.

Figure 1. (a) Experimental platform (b) Propulsion system

where p is the lag, which is the maximum truncation delay obtained by equation (2), $w_j(p)$is a weight function, which is optional and corresponds with the special choice of the Bartlett window, given by $w_j(p) = 1 - \frac{j}{p+1}$, which is a FIR filter*(Finite Impulse Response)*. The choice of lag p of the equation is an important characteristic of the KPSS test because, if chosen incorrectly, it can affect the efficiency of the test.

MATERIALS AND METHODS

The present research uses the deductive method, systematic observation and test execution as technic. Thus, the variables within the scope of this study are the current *i(k)*and the motor rotational speed*ω(k)*of the electromechanical propulsion system. The sampling time follows the Shannon/Nyquist theorem criterion, (Aguirre, 2007), so the value for the sampling interval (T_a) is 0.04s. For the development of this work, it was necessary to first collect the data. An experimental platform for electromechanical propellers was built at the Electrical Engineering laboratory of UNIJUI

Afterwards, the ADF, PP and KPSS tests were performed using the MATLAB software. The ADF test execution is done by the following command: [h; pValue; stat; cValue; reg] = adftest ('Y'; 'model'; 's'; 'lags'; 's').

Table 1: Physical characteristics of the experimental platform components

Components	Characteristics
Motor	Turnigy, model 2826/1400kw
Propeller	Dimensions 9x3.8'
Optical sensor	TC RT5000
ESC	RedBrick 30A
Current sensor	ACS 712
Battery	Lithium-polymer

In the same way, the PP test was executed by the command [h; pValue; stat; cValue; reg] = pptest('Y'; 'model'; 's'; 'lags'; 's'). And the KPSS test is executed by the command: [h; pValue; stat; cValue; reg] = kpsstest('Y'; 'lags'; 'v'; 'trend'; 'v'; 'alpha'; 'v'). It is noted here that Y is the data series that will be tested and the number of lags is obtained by equation 2, where N is substituted by 1300, which is the amount of analyzed data, resulting in 23. Both remarks are valid for all analyzed tests. For the ADF and PP test, the model will be the TS. It was chosen because the data has a linear trend and displacement. For the KPSS test, the “trend” structure was chosen because data the has a linear trend and is not a random ride. As the series has trend, the “true” option was used; otherwise, it would be the “false” option. “Alpha” indicates the significance level of the test, so it uses random values between 0.01 e 10. The tests execution provides the following information: *h:* In case it’s equal to 1, it indicates the rejection of the null hypothesis, while if it’s equal to 0, it indicates the null hypothesis; *pValue*: From the statistic defined for the test, *p* values are obtained by the data sampling. In this case, for the ADF and PP tests, the values are left-tailed probabilities on the distribution. Whereas for the KPSS test, the *p* values are right-tailed probabilities. *Stat*: The values of the test statistics, where the tests calculate its statistics using ordinary minimum least squares estimates of the coefficients on the alternative model. However, during the KPSS test performing, there’s a distinction due to the presence or not of the trend on the tested data. *cValue*: Critical values, are defined by the τ (tau) statistic distribution on the ADF and PP tests, and those are left-tailed probabilities on the distribution. On the KPSS test, the critical values are for right-tailed probabilities.

RESULTS AND DISCUSSION

Initially, the statistical parameters were calculated, as well as the average value and variance, and the results are presented on Table 2.

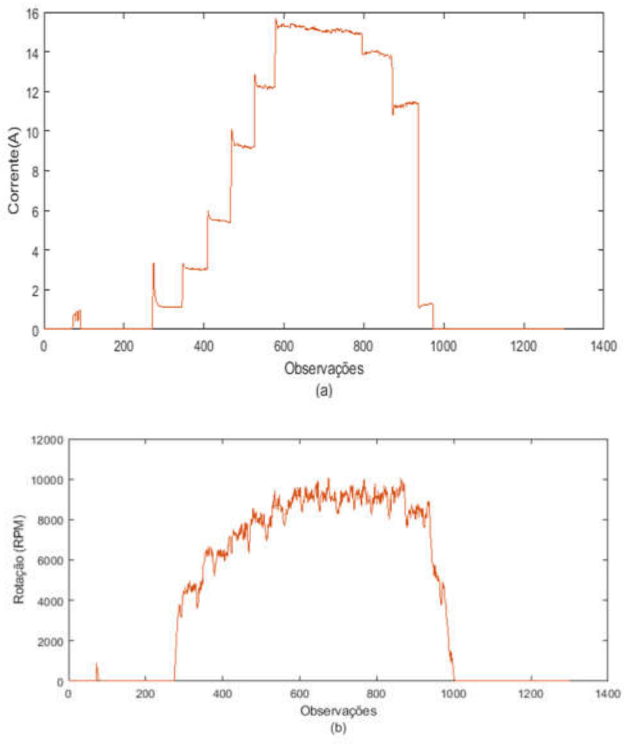

Figure 2. Dataset: (a) Electrical current *i(k)*, (b) Angular speed *ω(k)*.

Table 2: Statistical parameters of the variables

Parameters	Current (A) *i(k)*	Angular speed (RPM)*ω(k)*
Averagevalue	5.3285	4.2140e+03
Variance	39.4784	1.6411e+07

Table 3 presents the results obtained on the ADF, PP and KPSS tests execution, corresponding to the studied variables. Analyzing and observing table (3), it can be affirmed that the series are non-stationary. On the ADF and PP tests, a 95% assurance significance level is represented by the cValue parameter, which is -3.4141.

Table 3. Results for the tests of the studied variables

	Current	Electrical	i(k)	Speed	angular	w(k)
Test	ADF	PP	KPSS	ADF	PP	KPSS
h	0	0	1	0	0	1
PValue	0.9554	0.9526	0.0100	0.9489	0.9605	0.0100
Stat	-0.8876	-0.9128	1.1866	-0.9447	-0.8373	1.2700
CValue	-3.4144	-3.4144	0.2160	-3.4144	-3.4144	0.2160

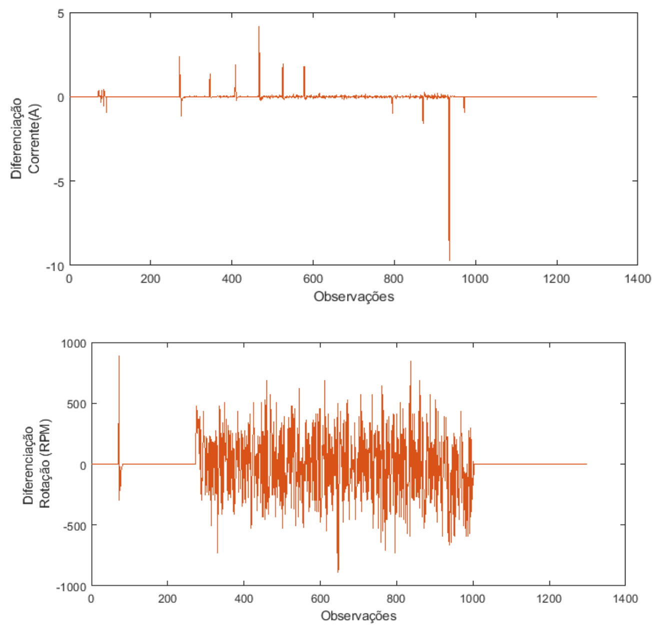

Figure 3. Data after differentiation: (a) Electrical current (b) Angular speed

It can also be observed that both the current and angular speed data present the stat (test statistic) valuegreater than the cValue (critical values) value. Thus, the null hypothesis is not rejected for both tests and consequently, both series are non-stationary, and, moreover, it can be seen that the h value is zero, as a confirmation. Observing and analyzing the KPSS test, the charted critical value of cValue= 0.2160 is obtained at 99%assurance. Similarly to the ADF and PP tests, the test statistic values are superior to the charted critical values in both data series. Therefore, according to this test’s theory, the series are non-stationary, because the null hypothesis is rejected, since h=1.

Thus, it is necessary to make the series stationary. According to what was proposed by Morretin and Toloi (2006), in order to perform differentiation, the MATLAB diff function was used. Figure 3 illustrates graphically the data series after the first differentiation. It can be noticed, by observing the graphs on figure (3), that they illustrate stationarity characteristics, but the confirmation was achieved by the reapplication of the ADF, KPSS and PP tests. The same command lines of the original series were used; the only difference is that the actual series used the differentiated data. Table 4 shows the results of the new tests. From the analysis of the results presented on table (4), it was noticed that the values found on the test statistic (stat) are smaller than their respective critical values in all tests and series.

Furthermore, as it is written, in order for the series to be stationary for the ADF and PP tests, the null hypothesis must be rejected, and for the KPSS test, the null hypothesis must be accepted; so it was confirmed, by observing the *h* values, that the series are stationary. On this work, the data series of current and angular speed were tested, verifying the presence of the unit root on them. Therefore, the criterion for choosing the most coherent mathematical representation were obtained, and consequently, more agility and effectiveness on modelling. It should be noted that the present work is a continuation of the research developed by VALER (2016), with the difference that they performed only the ADF and KPSS tests to confirm the presence of the unit root. Moreover, the dataset used was bigger when compared to the dataset used on this work.

Conclusion

With the application of the ADF, PP and KPSS tests to data contained on the time series of current $i(k)$ and angular speed $\omega(k)$, obtained from an electromechanical propeller, it can be verified whether there is violation of the statistical assumptions or not, and whether the obtained mathematical model is spurious or not. The use of the three tests contributed for increasing the assurance about the stationarity of the series. Therefore, the results allowed for the direct choice of an integrated auto-regressive model. The performed study is proposed as a method for the selection of the electromechanical propeller mathematical representation, because the assurance about non-stationarity, as well as the number of unit root present on the data series, give sustainability for the choice of the best mathematical representation for the behavioral dynamics of the system.

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