Available online at http://www.journalijdr.com

IJDR

ISSN: 2230-9926

International Journal of Development Research
Vol. 08, Issue, 03, pp.19534-19536, March, 2018

ORIGINAL RESEARCH ARTICLE

OPEN ACCESS

ON L-FUZZY TOPOLOGY FOR CONTINUOUS LATTICE

***Kamalesh Kumar Lal Karn**

Patan Multiple College, Lalitpur, India

ARTICLE INFO

Article History:

Received 09th December, 2017
Received in revised form
29th January, 2018
Accepted 08th February, 2018
Published online 30th March, 2018

Key Words:

Fuzzy Topology,
Lattice,
Scott Topology etc.

ABSTRACT

Fuzzy topology with respect to continuous lattice generalizes the classical theory. In this paper, an effort is being put to shed some light on important views regarding L-fuzzy topology for continuous lattice. Here, we look at the L-fuzzy topology for a continuous lattice L satisfying the infinite distributive law of meet over joints

Citation: Kamalesh Kumar Lal Karn, 2018. "on l-Fuzzy topology for continuous lattice", *International Journal of Development Research*, 8, (03), 19534-19536.

INTRODUCTION

For a general lattice L, the theory of L-fuzzy sets was introduced by Goguen. Fuzzy topology using a completely distributive lattice has been studied by fuzzy mathematicians and the term fuzzy lattice has been coined to describe such a lattice with an order reversing involution. Here we look at L-fuzzy topology for continuous lattice L satisfying infinite distributive law of meet over joins. No involution is required.

Continuous lattices

Let $(L, \subseteq)$ be a lattice with partial ordering $\subseteq$ and with meet and join denoted by $\cup$ *and* $\cap$ respectively. The suprimum and in fimum of subset X of L are written $\cup X$, $\cap X$ respectively.

Definition

A non empty set X in lattice L is directed if every finite subset of X has an upper bound in X i.e. for all x ,y ∈ X, there exists Z ∈ X with x $\subseteq z$, y $\subseteq z$

***Corresponding author: Kamalesh Kumar Lal Karn,**
Patan Multiple College, Lalitpur, India.

Definition

A complete lattice L is one in which every directed subset X $\subseteq L$, has a supremum. It follows classically that every subset of L has a supremum.

Definition

A fuzzy lattice is completely distributive lattice with an order reversing involution.

Definition

The Scott topology on (L, $\subseteq$) is defined as follows:

O $\subseteq$ L is open if

(i) x $\in$ O & x $\subseteq$ y $\Rightarrow$ y $\in$ O
(ii) $\cup X$ $\in$ O with X directed $\Rightarrow$ X $\cap$ O $\neq \phi$

i.e. there exists x $\in$ X such that x $\in$ O.

For further discussions lattices will always be considered with Scott topology &

L= (L, $\subseteq$) , L' = (L' , $\subseteq$) …

will range over complete lattices.

Proposition

L is a T_0 space which is in general not T_1.

Proposition

For lattices L, L', a function
F: L $\longrightarrow$ L' is a continuous iff F ($\cup$ X) = $\cup$ f(X) for all directed sets X $\subseteq$ L where f (X) = {f (X) : x $\in X$} & the second suprimum is in L'.

Proposition

The Cartesian product L x L' of complete lattices, partially ordered by

(x, x') $\subseteq$ (y, y') , iff x $\subseteq$ y & x' $\subseteq$ y', is a complete lattice with, for X $\subseteq$ L x L' ,
$\cup X = (\cup X_0 , \cup X_1)$

Where,

$X_0 = \{x \in L : \exists x' \in L' \text{ for which } (x, x') \in X\}$
& $X_1 = \{x' \in L' : \exists x \in L \text{ for which } (x, x') \in X\}$

Proposition

Let f: L x L' $\longrightarrow$ L'' Where L x L' has the Scott topology, then, f is continuous iff it is continuous on its argument separately i.e. iff for all $x_o \in$ L , $x'_0 \in$ L' , $f_{x'_0}$: L' $\longrightarrow$ L'' defined by ($f_{x'_0}$ (x) = f (x, x'_0)) & f_{x_0}: L $\longrightarrow$ L'' defined by (f_{x_0} (x') = f (x_0, x') are continuous.

Definition

For (x , y) $\in$ L , we say x is way below y (x << y) if y lies in the topological interior of the set {Z : x $\subseteq$ z} i.e. int {z $\in$ L : x $\subseteq$ z} = {z $\in$ L : x << z}

Definition

A continuous lattice is a complete lattice in which
X = $\cup$\{z $\in$ L : x << z\} for every x.

Proposition

For continuous lattice L, L' the scott topology & the product topology on the product L x L' coincide.

Example

The closed unit interval I = [0 , 1] is a complete lattice under the usual partial ordering $\leq$. The Scott topology is the right half open interval topology {[a, 1]: 0 $\leq$ a < 1 } then a << b iff a < b & (I, $\leq$) is a continuous lattice, which is in fact also fuzzy under the involution.

X $\longrightarrow$ 1 – x .

This argument can be extended to any complete chain.

- For any set S, the power set $\wp(S)$ is a fuzzy continuous lattice under set inclusion then for A, B $\subseteq$ S, A << B iff A is finite & A $\subseteq$ B.

Definition

An element x$\in$ L is compact if for every directed set X $\subseteq$ L , it follows that

$$x \subseteq \cup X \Rightarrow x \subseteq x_0 \text{ for some } x_0 \in X.$$

Remark:

Compact elements are isolated i.e. x << x. I has no compact while those of $\wp(S)$ are finite subsets of S.

Definition

A set E $\subseteq$ L is a basis for L if for all x$\in$ L,

x =$\cup$ { e : e $\in$ E & e $\in$ x} & $\cup$E' $\in$ E for all finite E' $\subseteq$ E.
If the second condition is absent , the set E is a sub basis. When E is countable, L is said to be countably based. Both I and $\wp(S)$ (S countable) are countably based, since bases are provided by the rationals & the finite subsets respectively.

L- Fuzzy Topology

Let (X, T) be a topological space & L be a continuous lattice with complete distributivity of meet over joints (i.e. a continuous frame).

For the set of functions f : X $\longrightarrow$ L pointwise partially ordered by f $\subseteq$ g iff f (x)($\subseteq$ g (x) for all x$\in$X ,

We have,

(f $\cap$ g) (x) = f (x) $\cap$ g (x)
& (f $\cup$ g) (x) = f (x) $\cup$ g (x)

Proposition

The set F of continuous functions from X to L (with the Scott topology) is closed under finite meets and arbitrary joins.

Proof:

We consider an arbitrary collection { f_i } of elements of F. The set {f_i (x) } together with all finite joins of its elements forms directed set z with

$\cup$Z = $\cup_i$ f_i (x).

For a given x $\in$ X , Let $\cup_i$ f_i (x) $\in$ O (Scott open).

Then by definition 1.1.4 (ii) ,

There exists a finite join say

f_i (x) ∪……………….∪ f_n (x) lying in O.
we assume f_i ∪……………………∪ f_n to be continuous.

Then there exists a nhd N_x of X such that

(f_i ∪…………… ∪f_n) (y) ∈ O for all y ∈ N_x .

But (f_i (y) U……∪ f_n ⊆ $\cup_I f_i$ (y)

So by definition 1.1.4 (i) ,

$\cup_I f_i$ (Y) ∈ O for all y ∈ N_x .
Hence $\cup_i$ f_i is continuous.
It only remains to show that F is closed under the finite meet and joins.

i.e. for f , g ∈ F.

f ∩ g , f ∪ g ∈ F.

we define h : X → L x L by

h (x) = (f(x) , g (x))
& m , j : L x L → L by

m (l_1 , l_2) = $l_1 \cup l_2$ respectively

Then (f ∩ g) (x) =(m∘h) (x).

While
(f ∪ g) (x) = (j∘ h) (x).

Now h is continuous with respect to the product topology on LxL , since f and g are continuous. And for the continuous lattice L, the product topology & the Scott topology on Lx L coincide. Finally m & j are known to be continuous with respect to the Scott topology on Lx L. We prove this latter assertion in order to show the necessity for one sided complete distributivity in L. By proposition 1.1.8, we need only to prove continuity on the arguments separately.

Writing J_{t_0} : L → L for the function defined by

J_{t_0} (l) = $\cup l_0$

then for X, a directed set

J_{t_0}(∪X) = (∪X) ∪ l_0
= { x ∪ l_0 : x ∈ X}
= ∪ J_{t_0} (x)

(ii) Defining now by M_{l_0} : L → L by M_{l_0} (l) = l ∩ l_0 ,

Then for X a directed set,

M_{l_0} (UX) = (UX) ∩ l_0
= ∪ {x l_0 : x ∈ X}
= ∪ M_{l_0} (X)

Conclusion

It is clear from 2.1 (ii), we conclude that it is held by complete distributivity of meet over joins. It is well known that the algebra of L- fuzzy sets reflect the properties of the lattice l , & that for L a fuzzy lattice, a theory of L – fuzzy topology can be constructed.

REFERENCES

Engelking, R. 1968. Outline of general topology, Amsterdam. Goguen, J. 1967. L- fuzzy sets, J. Math. Anal. Appl. 18. Azad, K. K. 1981.On fuzzy semi continuity, fuzzy alost continuity, and fuzzy weekly continuity, *J. Math. Anal. Appl.,* 82.

Michacl, E. 1957. Topology on space of subsets, *Trans. Amer, Math, soc*, 71.

Zadeh, L. 1965. Fuzzy sets, information and control 8. Monkre, J. R. 2008. A first course of Topology. Prasad, B. A thesis on study on separation axioms like properties in fuzzy topological space.

****