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STRONG AND WEAK VERTEX-EDGE MIXED DOMINATION ON S - VALUED GRAPHS

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ABSTRACT

In (Kiruthiga Deepa, 2016) we have defined the terms ev weight m dominating set, in which we have considered the weight of edges which dominates the weight of the vertices belonging to the spanning sub graph of $N_S(e)$. If we include the condition on the degrees of the edges and vertices belonging to the ev weight m dominating set, we obtain the notion of strong and weak ev weight m dominating set in G^S.

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INTRODUCTION

The study of domination in graph was initiated by Berge (1962). In (Rajkumar *et al*., 2015), the authors introduced the notion of semiring valued graphs. The domination in vertices and edges of an S-valued graph has been studied in (Jeyalakshmi *et al*., ?) and (Kiruthiga Deepa *et al*., 2017). Motivated by this we started studying mixed domination on S-valued graphs in (5) and (Kiruthiga Deepa, 2017). In (Kiruthiga Deepa, 2017), we have defined the term ev weight m dominating set, in which we have considered the weight of edges which dominates the weight of the vertices belonging to the spanning subgraph of $N_S(e)$. If we include the condition on the degrees of the edges and vertices 8belonging to the ev weight m dominating set, we obtain the notion of strong and weak ev weight m dominating set in G^S. In this paper we discuss the strong and weak ev weight m dominating set on G^S. In paper (Kiruthiga Deepa *et al*., 2017), on strong and weak ve weight m dominating set, we have proved that, for a minimal weak ve weight m dominating set having no isolate vertices in a vertex regular graph G^S, the complement is also a weak ve weight m dominating set. This result fails when we consider ev weight m dominating set which is a major result of this paper.

2. Preliminaries

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1: (Jonathan Golan, ?) A semi ring (S, +, .) is an algebraic system with a non-empty set S together with two binary operations + and . such that

- (S, +, 0) is a monoid.
- (S, .) is a semigroup.
- For all a, b, c $\in$ S, a . (b + c) = a. b + a. c and (a + b). c = a. c + b. c
- 0. x = x. 0 = 0, $\forall$ x $\in$ S.

Definition 2.2: (Jonathan Golan, ?) Let (S, +, .) be a semiring. A Canonical Pre-order $\preceq$ in S defined as follows: for a, b $\in$ S , a $\preceq$ b if and only if, there exists an element c $\in$ S such that a + c = b.

Definition 2.3: (Rajkumar *et al.*, 2015) Let $G = (V, E \subset V \times V)$ be a given graph with $V, E \neq \phi$. For any semiring (S, +, .), a semi ring-valued graph (or a S-valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be $\psi(x, y) = \begin{cases} \min\{\sigma(x), \sigma(y)\}, \text{ if } \sigma(x) \preceq \sigma(y) \text{ or } \sigma(y) \preceq \sigma(x) \\ 0, \text{otherwise} \end{cases}$

For every unordered pair (x,y) of $E \subset V \times V$. We call σ, a S- vertex set and ψ, a S-edge set of G^S.

Definition2.4: (Rajkumar *et al.*, 2015) The degree of the vertex v_i of the S- valued graph G^S is defined as

$\deg_S(v_i) = \left(\sum_{(v_i, v_j) \in E} \psi((v_i, v_j)), l \right)$ where l is the number of edges incident with v_i .

Definition2.5: (Mangala Lavanya *et al.*, 2016) Let $G^S = (V, E, \sigma, \psi)$ be a S- valued graph. The degree of the edge e is defined as $\deg_S(e) = \left(\sum_{e_i \in N_S(e)} \psi((e_i)), m \right)$ where m is the number of edges adjacent to e.

Definition 2.6: (Jeyalakshmi *et al.*, 2015) A S- valued graph $G^S = (V, E, \sigma, \psi)$ is said to be a S-Star if its underlying graph G is a Star along with S-values.

Definition 2.7: (Mangala Lavanya, 2017) Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ E is said to be a strong weight dominating edge set if

- D is a weight dominating edge set.
- For each edge e $\in$D, $\deg_S(e_i) \preceq \deg_S(e) \forall e_i \in N_S[e]$

Definition 2.8: (Mangala Lavanya, 2017) Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ E is said to be a weak weight dominating edge set if

- D is a weight dominating edge set.
- For each edge e $\in$D, $\deg_S(e) \preceq \deg_S(e_i) \forall e_i \in N_S[e]$

Definition 2.9: (Kiruthiga Deepa, 2017) Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. An edge e $\in$ E is said to be a ev weight m dominating edge of a vertex v, if $\sigma(v) \preceq \psi(e), \forall v \in \langle N_S[e] \rangle$.

Definition 2.10: (Kiruthiga Deepa, 2017) Consider the S valued graph $G^S = (V, E, \sigma, \psi)$.Let T $\subseteq$ E. If every vertex of G^S is weight m dominated by any edge in T, then T is said to be a ev weight m dominating set.

3. Strong and Weak Edge – Vertex Mixed Domination on S-Valued Graphs

In this section, we introduce the notion of strong and weak edge - vertex mixed domination in S valued graph and prove some simple results. In paper (Kiruthiga Deepa, 2017), we have defined the terms ev weight m dominating set, in which we have considered the weight of edges which dominates the weight of the vertices belonging to the spanning subgraph of $N_S(e)$. If we include the condition on the degrees of the edges and vertices belonging to the ev weight m dominating set, we obtain the following definitions.

Definition 3.1: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T $\subseteq$ E is said to be a strong ev weight m dominating set, if

- T is a ev weight m dominating set.
- For each edge e ∈ T, $\deg_S(v_i) \preceq \deg_S(e) \forall v_i \in \langle N_S[e] \rangle$.

Example 3.2: Let (S = {0, a, b, c}, +, .) be a semiring with the following Cayley Tables:

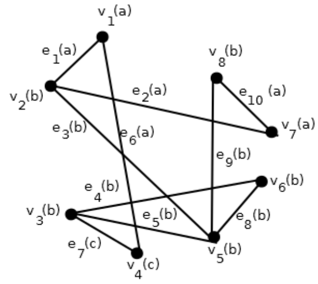
+	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

.	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	C

Let $\preceq$ be a canonical pre-order in S, given by

0 $\preceq$ 0, 0 $\preceq$ a, 0 $\preceq$ b, 0 $\preceq$ c, a $\preceq$ a, a $\preceq$ b, a $\preceq$ c, b $\preceq$ b, c $\preceq$ b, c $\preceq$ c

Consider the S - graph $G^S = (V, E, \sigma, \psi)$,



where $\sigma : V \to S$ is defined by

σ(v1) = σ(v7) = a, σ(v4) = c, σ(v2) = σ(v3) = σ(v5) = σ(v6) = σ(v8) = b

and ψ : E → S is defined by

ψ(e1) = ψ(e2) = ψ(e6) = ψ(e10) = a, ψ(e7) = c, ψ(e3) = ψ(e4) = ψ(e5) = ψ(e8) = ψ(e9) = b

Clearly T = {e3, e5} is a strong ev weight m dominating set.

Similarly T1= {e3, e5, e8}, T2 = {e3, e5, e9}, T3 = {e3, e5, e8, e9} are strong ev weight m dominating sets.

Definition 3.3: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T ⊆ E is said to be a minimal strong ev weight m dominating set, if

- T is a strong ev weight m dominating set.
- No proper subset of T is a strong ev weight m dominating set.

In the example 3.2, T = {e3, e5} is a minimal strong ev weight m dominating set.

Definition 3.4: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T ⊆ E is said to be a maximal strong ev weight m dominating set, if

- T is a strong ev weight m dominating set.
- there is no strong ev weight m dominating set T' ⊂ E such that T ⊂ T' ⊂ E.

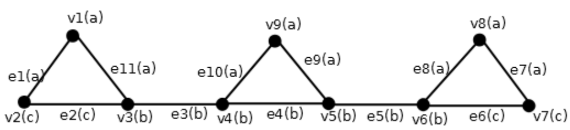
In the example 3.2, T3 = {e3, e5, e8, e9} is a maximal strong ev weight m dominating set.

Definition 3.5: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T ⊆ E is said to be a strong ev weight m dominating independent set, if

- T is a strong ev weight m dominating set.
- if e, f∈T then $N_S(e) \cap (f, \psi(f)) = \varphi$.

Example 3.6: Let (S = {0, a, b, c}, +, ·) be a semiring with the canonical preorder given in example 3.2

Consider the S valued graph G^S :



Define σ: V → S by

σ(v1) = σ(v9) = σ(v8) = a, σ(v3) = σ(v4) = σ(v5) = σ(v6) = b, σ(v2) = σ(v7) = c

and ψ: E → S by
ψ(e1) = ψ(e7) = ψ(e8) = ψ(e9) = ψ(e10) = ψ(e11) = a, ψ(e2) = ψ(e6) = c, ψ(e3) = ψ(e4) = ψ(e5) = b
Clearly T = {e3, e5} is a strong ev weight m dominating independent set.

Definition 3.7: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T ⊆ E is said to be a weak ev weight m dominating set, if

- T is a ev weight m dominating set.
- For each edge e ∈ T, $\deg_S(e) \preceq \deg_S(v_i) \forall v_i \in \langle N_S[e] \rangle$.

Example 3.8: Let (S = {0, a, b, c}, +, .) be a semiring with the canonical preorder given in example 3.2
Consider the S valued graph $G^S = (V, E, \sigma, \psi)$

Define σ: V → S by
σ(v1) = σ(v2) = σ(v5) = σ(v6) = b, σ(v3) = a, σ(v4) = c.
and ψ : E → S by
ψ(e1) = ψ(e5) = b, ψ(e2) = ψ(e3) = ψ(e7) = a, ψ(e4) = ψ(e6) = c.
Clearly T = {e1, e5} is a weak ev weight m dominating set.

Definition 3.9: Consider the S valued graph $G^S = (V, E, \sigma, \psi)$. A subset T ⊆ E is said to be a weak ev weight m dominating independent set, if

- T is a weak ev weight m dominating set.
- if e, f∈T then $N_S(e) \cap (f, \psi(f)) = \varphi$.

In the e x a m p l e 3.8, T= {e1, e5} is a weak ev weight m dominating independent set, since
$N_S(e1) \cap (e5, b) = \varphi$.

Theorem 3.10: In a S-Wheel, the minimal strong ev weight m dominating set exist and is unique, provided the edge is a spoke.
Proof: Let G^S be a S-Wheel. Let e1 be any spoke of G^S with maximum weight.

Then $v_i \in \langle N_S[e_1] \rangle, \forall v_i \in G^S$. Also $\deg_S(v_i) \preceq \deg_S(e_1) \forall v_i \in \langle N_S[e_1] \rangle$.
∴ {e1} is the strong ev weight m dominating set.

Hence the minimal strong ev weight m dominating set is unique.

Remark 3.11: The above theorem also holds for a Complete graph and a Complete Bipartite Graph.

- In a Complete graph K_n^S the minimal strong ev weight m dominating set is unique.
- In a Complete Bipartite graph $K_{m,n}^S$,the minimal strong ev weight m dominating set is unique.

Remark 3.12: In a S Star, there will be no strong ev weight m dominating set, since the pole has maximum degree than all edges and every edge is connected to the pole.

Theorem 3.13: A strong ev weight m dominating set T of a graph G^S is a minimal strong ev weight m dominating set of G^S iff every edge e ∈ T satisfies at least one of the following properties:

- there exist an edge f∈E T, such that $N_S(f) \cap (T \times S) = \{(e, \psi(e))\}$.
- e is adjacent to no edge of T.

Proof: Let e ∈ T. Assume that e is adjacent to no edge of T, then T {e} cannot be a strong ev weight m dominating set.
⇒ T is a minimal strong ev weight m dominating set.

On the other hand, if for any e ∈ T, there exist a f ∈ E T such that $N_S(f) \cap (T \times S) = \{(e, \psi(e))\}$

Then f is adjacent to e ∈ T and no other edge of T. In this case also, T {e} cannot be a strong ev weight m dominating set of G^S. Hence T is a minimal strong ev weight m dominating set.

Conversely, assume that T is a minimal strong ev weight m dominating set of G^S. Then for each e ∈ T, T {e} is not a strong ev weight m dominating set of G^S.

∴ there exist an edge, f ∈ E (T {e}) that is adjacent to no edge of (T {e}).
If f = e, then e is adjacent to no edge of T.
If f ≠ e, then T is a strong ev weight m dominating set and f ∉ T ⇒ f is adjacent to at least one edge of T. However f is not adjacent to any edge of T {e}. ⇒ N_S (f) ∩ T × S = {(e, ψ(e))}.

Theorem 3.14: A subset T ⊆ E of G^S is a strong ev weight m dominating independent set iff T is a maximal strong independent edge set in G^S.

Proof: Clearly every maximal strong independent edge set T in G^S is a strong ev-weight m-dominating independent set.

Conversely, assume that T is a strong ev weight m dominating independent set. Then T is independent and every edge not in T is adjacent to an edge of T and therefore T is a maximal strong independent edge set in G^S.

Theorem 3.15: Every maximal strong independent edge set of G^S is a minimal strong ev weight m dominating set.

Proof: Let T be a maximal strong independent edge set of G^S. Then by theorem 3.14, T is a strong ev weight m dominating set.

Since T is independent, every edge of T is adjacent to no edge of T. Thus, every edge of T satisfies the second condition of theorem 3.13. Hence T is a minimal strong ev weight m dominating set in G^S. Combining the above two theorems, we obtain the following theorem,

Theorem 3.16: A subset T ⊆ E of G^S is a strong ev weight m dominating independent set iff T is a minimal strong ev weight m dominating set.

The following theorem is obvious.

Theorem 3.17: A subset T ⊆ E of G^S is a weak ev weight m dominating independent set iff T is a weak ev weight m dominating set.

Remark 3.18:

- In a S Star, there will be no weak ev weight m dominating set, since except the pole every vertex has minimum degree than all the edges.
- In a S Wheel, there will be no weak ev weight m dominating set, since all the vertices has either same or minimum degree than all the edges.
- In a Complete graph, there will be no weak ev weight m dominating set, since every vertex has minimum degree than all the edges.
- In a Complete Bipartite graph, there will be no weak ev weight m dominating set, since every vertex has minimum degree than all the edges.

4. Conclusion

(1) In (4), we observe that in a S-valued graph G^S, a strong ve weight m dominating set is unique if it exists.

(2) In this paper we observe that in a S-valued graph G^S, a weak ev weight m dominating set is unique if it exists.
In our paper (5), we have proved

(**Theorem 4.8**): If D ⊆ V of G^S is a minimal weak ve weight m dominating set without S isolate vertices then V D is also a weak ve weight m dominating set of G^S, whenever G^S is vertex regular S valued graph.

(1) The above result does not exist for a weak ev weight m dominating set, as G^S has no minimal weak ev weight m dominating set.

(2) Even though the minimal strong ev weight m dominating set exists, the above result does not exist, since the complement of a strong ev weight m dominating set will not be a strong ev weight m dominating set. In the example 3.2, T = $\{e_3, e_5\}$ is a minimal strong ev weight m dominating set. Here E T = $\{e_1, e_2, e_4, e_6, e_7, e_8, e_9, e_{10}\}$ is not a minimal strong ev weight m dominating set.

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