Available online at http://www.journalijdr.com

IJDR

ISSN: 2230-9926

International Journal of Development Research

Vol. 07, Issue, 10, pp.15988-15995, October, 2017

ORIGINAL RESEARCH ARTICLE OPEN ACCESS

FUZZY INVENTORY MODEL FOR TIME DEPENDENT DETERIORATING ITEMS WITH LEAD TIME STOCK DEPENDENT DEMAND RATE AND SHORTAGES

*Palani, R. and Maragatham, M.

Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli - 620023, Tamil Nadu, India

ARTICLE INFO

Article History:

Received 19th July, 2017
Received in revised form
24th August, 2017
Accepted 17th September, 2017
Published online 10th October, 2017

Keywords:

Fuzzy inventory model, Fuzzy number, deteriorating items,
Stock dependent demand rate, Lead time.

***Corresponding author**

ABSTRACT

In this paper a deterministic fuzzy inventory model for time dependent deteriorating items with lead time and stock dependent demand rate has been developed. The ordering cost, deterioration cost, holding cost and shortage cost are assumed as triangular fuzzy number. In this model shortages are allowed during the lead time and completely backlogged. An economic order quantity (EOQ) has been obtained. The derived model is illustrated by a numerical example.

Citation: Palani, R. and Maragatham, M. 2017. "Fuzzy Inventory Model for Time Dependent Deteriorating Items with Lead Time Stock Dependent Demand Rate and Shortages", *International Journal of Development Research*, 7, (10), 15988-15995.

INTRODUCTION

It is usually that a large quantity of goods on self in a superstore will lead the customer to buy more goods and that situation creates the greater demand of the goods. This situation motivates the retailer to increase their order quantity. Deterioration of physical goods is one of the important factors in any inventory system. Inventory is a part of each and every organization, whether it is manufacturing or service organization. All organizations have to keep some inventory for smooth running of their business. If any organization claims that they are not keeping any inventory it's an absurd. The Economic Order quantity model is one of the oldest known model developed by Wilson (1934), Rekha, Vikas, Urvashi (2013) and a lot of work has been done on this model. In the last few decades the study of perishable items has gained enormous importance. In present scenario the wastage of resources is considered as a sin. Even most of the companies are facing cut throat competition and deterioration of resources would reduce their profit margins drastically. Therefore, in most of the present models the items considered are deteriorating items and inventory cost compromises of the deterioration cost. Ghare and Schrader were the pioneer to use the concept of deterioration, they developed an inventory model with a constant rate of deterioration Ghare and Schrader (1963), followed by Covert and G. C. Philip (1973) who formulated a model considering a variable rate of deterioration with two parameter Weibull distribution, which was further extended by Pal, S. (1993), Nahmais (1982) provided the relevant literature on the problem of determining suitable ordering policies for both fixed life perishable inventory, and inventory subject to continuous exponential decay. The Japanese experience of using Just-In-Time (JIT) production shows that there are advantages and benefits associated with their efforts to control lead time. Japanese manufacturers are known for their strong and lasting partnership with their suppliers. This helps reduce lead time and is one of the sources of success of their JIT philosophy.

Lead time has been a topic of interest for many authors Ben-daya (1994), Das (1975), Foote (1988), Magson (1979), Naddor (1966), Chung, and Ting (1993), Fujiwara (1993). Almost all authors assume lead time as prescribed in all cases, i.e. deterministic as well as probabilistic. However, in many practical situations lead time can be reduced at an added cost. By reducing the lead time, customer service and responsiveness to production schedule changes can be improved and reduction in safety stocks can be achieved. The added cost of reducing lead time consists mainly of administrative costs, transportation cost as the item's transit time from the supplier is a major component of lead time, and supplier's speed-up cost.

However, in certain situations, uncertainties are due to fuzziness, and such cases are dilated in the fuzzy set theory which was demonstrated by Zadeh (1965). Kaufmann and Gupta (1991) provided an introduction to fuzzy arithmetic operation and Zimmermann (1996) discussed the concept of the fuzzy set theory and its applications. Considering the fuzzy set theory in inventory modeling renders an authenticity to the model formulated since fuzziness is the closest possible approach to reality. Park (1987) applied the fuzzy set concepts to EOQ formula by representing the inventory carrying cost with a fuzzy number and solved the economic order quantity model using fuzzy number operations based on the extension principle. Vujosevic, Petrovic, and Petrovic (1996) used trapezoidal fuzzy number to fuzzify the order cost in the total cost of the inventory model without backorder, and got fuzzy total cost. Yao and Lee (1996) developed a backorder inventory model with fuzzy order quantity as triangular and trapezoidal fuzzy numbers and shortage cost as a crisp parameter. Gen, Tsujimura, and Zheng (1997) determined their input data as fuzzy numbers, and then the interval mean value concept was introduced to solve the inventory problem. Chang, Yao, and Lee (1998) determined the backorder inventory problem with fuzzy backorder such that the backorder quantity is a triangular fuzzy number. Chang (1999) developed the fuzzy production inventory model for fuzzify the product quantity as triangular fuzzy number. Yao, Chang, and Su (2000) assumed to be the order quantity and the total demand rate as triangular fuzzy numbers and obtained the fuzzy inventory model without shortages. Yao and Chiang (2003) considered the total cost of inventory without backorder. They fuzzified the total demand and cost of storing one unit per day into triangular fuzzy numbers and defuzzify by the centroid and the signed distance methods. Chang, Yao, and Ouyang (2006) determined the mixture inventory model involving variable lead-time with backorders and lost sales. First, they fuzzify the random lead-time demand to be a fuzzy random variable and then fuzzify the total demand to be the triangular fuzzy number and derive the fuzzy total cost. By the centroid method of defuzzification, they estimated the total cost in the fuzzy sense. Wee, Yu, and Chen (2007) introduced an optimal inventory model for items with imperfect quality and shortage backordering. Lin (2008) developed the inventory problem for a periodic review model with variable lead-time and fuzzified the expected demand shortage and backorder rate using signed distance method to defuzzify. Roy and Samanta (2009) discussed a fuzzy continuous review inventory model without backorder for deteriorating items in which the cycle time is taken as a symmetric fuzzy number. They used the signed distance method to fuzzify the total cost. Gani and Maheswari (2010) developed an EOQ model with imperfect quality items with shortages where defective rate, demand, holding cost, ordering cost and shortage cost are taken as triangular fuzzy numbers. Graded mean integration method is used for defuzzification of the total profit. Ameli, Mirzazadeh, and Shirazi (2011) developed a new inventory model to determine ordering policy for imperfect items with fuzzy defective percentage under fuzzy discounting and inflationary conditions.

They used the signed distance method of defuzzification to estimate the value of total profit. Sadi-Nezhad, Memar Nahavandi, and Nazemi (2011) developed a periodic review model and a continuous review inventory model with fuzzy setup cost, holding cost and shortage cost. They also considered the lead-time demand and the lead-time plus one period's demand as random variables. They use two methods in the name of signed distance and possibility mean value to defuzzify. Uthayakumar and Valliathal (2011) developed an economic production model for Weibull deteriorating items over an infinite horizon under fuzzy environment and considered some cost component as triangular fuzzy numbers and using the signed distance method to defuzzify the cost function. Kumar *et al*. developed a fuzzy inventory model with limited storage capacity. Kumari, Kumar, and Singh (2013) investigate a fuzzy two ware house inventory model with three component demand rate. Tayal, Singh, and Sharma (2014) determined an inventory model for deteriorating items with seasonal products and an option of an alternative market. Kumar and Rajput (2015) introduced Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging. Kumar and Kumar (2016) introduced an inventory model with stock-dependent demand rate for deterioration items. Recently Kumar and Kumar (2016a) presented an Inventory Model for deteriorating items stock dependent demand and partial backlogging. Recently, R. Palani (2017), *et al* developed Fuzzy EOP model for controllable deterioration rate and time dependent demand and inventory holding cost. R. Palani (2017a), et.al presented Fuzzy EOQ model with shortages for products with controllable deterioration rate and time dependent demand and inventory holding cost. In this paper, a fuzzy deterministic inventory model for time dependent deteriorating items with lead time and stock dependent demand rate has been developed. We extend the Maragatham and Palani model by considering time dependent deterioration rate and stock dependent demand rate. In this model an economic order quantity has been obtained. The numerical example is presented to illustrate the proposed model.

Preliminaries

Basic Definitions

Fuzzy Set

If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x; \mu_{\tilde{A}}(x))/x\epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set $\tilde{A}$. The membership function maps each element of X to a membership grade between o and 1 (included).

α - Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree of α is called the α level set or α - cut $(i.e)$, $\tilde{A}^{(\alpha)} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$.

Fuzzy Number

Fuzzy numbers are of great important in fuzzy systems.

Fuzzy Number

A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}: R \to [0,1]$ is called a fuzzy number if

- $\tilde{A}$ is normal, $(i.e)$, there exit an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$.
- $\tilde{A}$ is fuzzy convex,
- $(i.e)$, $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)\ x_1, x_2 \in R, \forall\ \lambda \in [0,1]$.
- $\mu_{\tilde{A}}$ is upper continuous.
- $supp\ \tilde{A}$ is bounded,
- where $supp\ \tilde{A} = \{x \in R: \mu_{\tilde{A}}(x) > 0\}$.

Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions, is a generalized fuzzy number

- $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.
- $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \leq a_1$.
- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$.
- $\mu_{\tilde{A}}(x) = 1$, $a_2 \leq x \leq a_3$.
- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$.
- $\mu_{\tilde{A}}(x) = 0$, $a_4 \leq x \leq \infty$, where a_1, a_2, a_3, a_4 are real numbers.

Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$, where $a_1 \leq a_2 \leq a_3$ and defined on R, is called triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & otherwise \end{cases}$$

Operations of Triangular Fuzzy Number

Consider two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$, $\tilde{B} = (b_1, b_2, b_3)$.

i. The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\begin{aligned} \tilde{A} + \tilde{B} &= (a_1, a_2, a_3) + (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \end{aligned}$$

where a_1, a_2, a_3, b_1, b_2, b_3 are real numbers.

ii. The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$,

where $T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$, $c_1 = \min T$, $c_2 = a_2 b_2$, $c_3 = \max T$

If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

iii. $[-\tilde{B}] = -(b_1,b_2,b_3) = (-b_3,-b_2,-b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is

$$\tilde{A} - \tilde{B} = (a_1,a_2,a_3) - (b_1,b_2,b_3)$$
$$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3 are all non zero positive real numbers, then division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = \frac{(a_1,a_2,a_3)}{(b_1,b_2,b_3)} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

v. For any real number

$$k, k\tilde{A} = \begin{cases}(ka_1,ka_2,ka_3), & if\ k > 0\\ (ka_3,ka_2,ka_1), & if\ k < 0\end{cases}$$

Defuzzification

Defuzzification is the conversion of a fuzzy quantity to a crisp quantity. Defuzzification methods obtain the representative value of a fuzzy set.

Graded Mean Representation Method

Let $\tilde{A}$ be a fuzzy number with left reference function L and right reference function R. Let L^{-1} and R^{-1} be the inverse functions of L and R respectively.

The graded mean integration representation of $(\tilde{A})$ is defined by

$$\rho(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 h[L^{-1}(h) + R^{-1}(h)]dh}{\int_0^1 h\,dh}$$ with 0<h<1

By the above formula, the graded mean representations of triangular fuzzy number $\tilde{A} = (a_1,a_2,a_3)$ is given by $(\tilde{A}) = \frac{a_1+4a_2+a_3}{6}$.

Notations and assumptions

The following notations are used in developing the model.

A_D : Amount of material deterioration during a cycle time.
$\theta(t)$: Time dependent Deterioration rate.
$\tilde{c}$: Fuzzy unit cost per item
$\tilde{A}$: Fuzzy ordering cost of inventory/order
$D(t)$: The demand rate (units per unit time)
t_1 : Replenishment cycle time
L : Lead time
P_c : Purchase cost
$\tilde{S}_c$: Fuzzy Shortage cost
C_H : The total cost of holding inventory per cycle
C_D : Total deterioration cost per cycle
Q : Maximum Inventory Level
$\tilde{h}$: Fuzzy inventory holding cost per unit item per unit time.
$I(t)$: The inventory level at time t

Assumptions

I adopt the following assumptions and notations for the model to be discussed.

- The demand $D(t)$ is stock dependent in linear form $D(t) = a + bI(t)$.
- The item cost remains constant irrespective of the order size.

- Shortages are allowed.
- Replenishment rate is infinite and the lead time is constant.
- The holding cost is constant.
- The items considered are deteriorating items but deterioration is not instantaneous.
- The deterioration rate $\theta(t)$ is dependent on time in linear form is $\beta(t) = bt$.
- There is no repair or replenishment of the deteriorated items during the inventory cycle.
- The inventory is replenished only once in each cycle.
- During lead time shortages are allowed
- Ordering quantity is $Q + LD(t)$ when $t = L$.

Mathematical model and Analysis

In this model deterministic demand is considered stock dependent and time dependent rate of deterioration, depletion of the inventory occurs due to demand (supply) as well as due to deterioration in each cycle. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the holding cost constant and shortages are allowed. The behavior of inventory system at any time is shown in figure 1**.**

The states of $I(t)$ over the cycle time T is given by the following first order differential equation.

$$\frac{dI(t)}{dt} + \beta t I(t) = -(a + bI(t)), \quad L \le t \le t_1 \tag{1}$$

$$I(t).e^{bt+\frac{\beta}{2}t^2} = -a\left[t + \frac{\beta}{2}t^2 + \frac{\beta}{6}t^3\right] + K \tag{2}$$

Where K is constant of integration

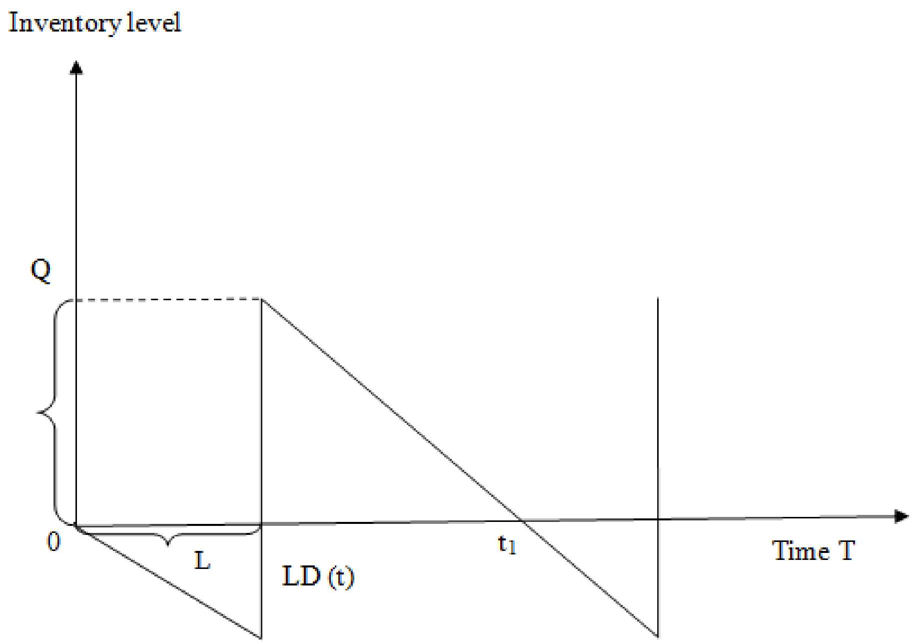

Figure 1. Graphical representation of the inventory system

Solving equation (1), we obtain $I(t)$ during the time period $(L \le t \le t_1)$

$$I(t) = \left[1 - bt - \frac{\beta}{2}t^2\right]\left[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{\beta}{6}(t_1^3 - t^3)\right], \quad L \le t \le t_1 \tag{3}$$

At time $t = L,\ I(L) = Q$ i.e. when the items are received, the level at which the organization is having a maximum inventory and equation (3) give the value of Q where $Q + LD(t)$ is the quantity ordered at the start of the cycle. As it is assumed there is lead time, i.e. items are received non-instantaneously.

$$Q = \left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right] \tag{4}$$

Since the total demand during cycle period t_1 is $D(t)(t_1 - L)$, the amount of materials which deteriorates during one cycle is

$$A_D = \left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}[1 - b(t_1 - L)] - a(t_1 - L) \quad (5)$$

The total variable cost will consist of the following costs

(a) The ordering cost of the materials, which is fixed per order for the present financial year.

(b) The deterioration cost is given by $c.A_D$ which comes out to be

$$D_c = c\left\{a(t_1 - L) + b(t_1 - L)\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right\} \quad (6)$$

(c) The holding cost is the function of average inventory cost and it is given by

$$\int_L^{t_1} h\, I(t)dt$$

Which, upon simplification, yields

$$C_H = h\left\{\left[a\left(\frac{t_1^2}{2}\right) + b\left(\frac{t_1^3}{2}\right) + \frac{\beta}{8}(t_1^4) - \frac{ab}{6}(t_1^3) - \frac{b^2}{8}(t_1^4) - \frac{b\beta}{20}(t_1^5) + \frac{a\beta}{24}(t_1^4) + \frac{b\beta}{15}(t_1^5) + \frac{\beta^2}{72}(t_1^6)\right] - \left[a\left(Lt_1 - \frac{L^2}{2}\right) + \frac{b}{2}\left(Lt_1^2 - \frac{L^3}{3}\right) + \frac{\beta}{6}\left(Lt_1^3 - \frac{L^4}{4}\right) - ab\left(\frac{L^2}{2}t_1 - \frac{L^3}{3}\right) - \frac{b^2}{2}\left(\frac{L^2}{2}t_1^2 - \frac{L^4}{4}\right) - \frac{b\beta}{6}\left(\frac{L^2}{2}t_1^3 - \frac{L^5}{5}\right) + \frac{a\beta}{2}\left(\frac{L^3}{3}t_1 - \frac{L^4}{4}\right) + \frac{b\beta}{2}\left(\frac{L^3}{3}t_1^2 - \frac{L^5}{5}\right) + \frac{\beta^2}{12}\left(\frac{L^3}{3}t_1^3 - \frac{L^6}{6}\right)\right]\right\} \quad (7)$$

(d) The purchase cost is given by

$$P_c = c[Q + L(a + bI(t))]$$

$$P_c = c\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right] + \mathrm{L}\left[\mathrm{a} + \mathrm{b}\left\{\left[1 - \mathrm{bL} - \frac{\beta}{2}\mathrm{L}^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right]\right\} \quad (8)$$

(e) Shortage cost is given by $S_c = S\{L[a + bI(t)]\}$

$$S_c = S\left\{L\left[a + b\left[\left\{1 - bL - \frac{\beta}{2}L^2\right\}\left\{a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right\}\right]\right]\right\} \quad (9)$$

Total variable cost function for one cycle is given by

$$TC = OC + P_c + H_c + D_c + S_c$$

$$TC = A + c\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right] + \mathrm{L}\left[\mathrm{a} + \mathrm{b}\left\{\left[1 - \mathrm{bL} - \frac{\beta}{2}\mathrm{L}^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right]\right\} + h\left\{\left[a\left(\frac{t_1^2}{2}\right) + b\left(\frac{t_1^3}{2}\right) + \frac{\beta}{8}(t_1^4) - \frac{ab}{6}(t_1^3) - \frac{b^2}{8}(t_1^4) - \frac{b\beta}{20}(t_1^5) + \frac{a\beta}{24}(t_1^4) + \frac{b\beta}{15}(t_1^5) + \frac{\beta^2}{72}(t_1^6)\right] - \left[a\left(Lt_1 - \frac{L^2}{2}\right) + \frac{b}{2}\left(Lt_1^2 - \frac{L^3}{3}\right) + \frac{\beta}{6}\left(Lt_1^3 - \frac{L^4}{4}\right) - ab\left(\frac{L^2}{2}t_1 - \frac{L^3}{3}\right) - \frac{b^2}{2}\left(\frac{L^2}{2}t_1^2 - \frac{L^4}{4}\right) - \frac{b\beta}{6}\left(\frac{L^2}{2}t_1^3 - \frac{L^5}{5}\right) + \frac{a\beta}{2}\left(\frac{L^3}{3}t_1 - \frac{L^4}{4}\right) + \frac{b\beta}{2}\left(\frac{L^3}{3}t_1^2 - \frac{L^5}{5}\right) + \frac{\beta^2}{12}\left(\frac{L^3}{3}t_1^3 - \frac{L^6}{6}\right)\right]\right\} + c\left\{a(t_1 - L) + b(t_1 - L)\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right\} + S\left\{L\left[a + b\left[\left\{1 - bL - \frac{\beta}{2}L^2\right\}\left\{a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right\}\right]\right]\right\} \quad (10)$$

Hence the fuzzy total cost per unit time is given by

$$\widetilde{TC} = \widetilde{OC} + P_c + \widetilde{H_c} + \widetilde{D_c} + \widetilde{S_c}$$

$$\widetilde{TC} = \tilde{A} + c\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right] + \mathrm{L}\left[\mathrm{a} + \mathrm{b}\left\{\left[1 - \mathrm{bL} - \frac{\beta}{2}\mathrm{L}^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right]\right\} + \tilde{h}\left\{\left[a\left(\frac{t_1^2}{2}\right) + b\left(\frac{t_1^3}{2}\right) + \frac{\beta}{8}(t_1^4) - \frac{ab}{6}(t_1^3) - \frac{b^2}{8}(t_1^4) - \frac{b\beta}{20}(t_1^5) + \frac{a\beta}{24}(t_1^4) + \frac{b\beta}{15}(t_1^5) + \frac{\beta^2}{72}(t_1^6)\right] - \left[a\left(Lt_1 - \frac{L^2}{2}\right) + \frac{b}{2}\left(Lt_1^2 - \frac{L^3}{3}\right) + \frac{\beta}{6}\left(Lt_1^3 - \frac{L^4}{4}\right) - ab\left(\frac{L^2}{2}t_1 - \frac{L^3}{3}\right) - \frac{b^2}{2}\left(\frac{L^2}{2}t_1^2 - \frac{L^4}{4}\right) - \frac{b\beta}{6}\left(\frac{L^2}{2}t_1^3 - \frac{L^5}{5}\right) + \frac{a\beta}{2}\left(\frac{L^3}{3}t_1 - \frac{L^4}{4}\right) + \frac{b\beta}{2}\left(\frac{L^3}{3}t_1^2 - \frac{L^5}{5}\right) + \frac{\beta^2}{12}\left(\frac{L^3}{3}t_1^3 - \frac{L^6}{6}\right)\right]\right\} + \tilde{c}\left\{a(t_1 - L) + b(t_1 - L)\left\{\left[1 - bL - \frac{\beta}{2}L^2\right]\left[a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right]\right\}\right\} + \tilde{S}\left\{L\left[a + b\left[\left\{1 - bL - \frac{\beta}{2}L^2\right\}\left\{a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2) + \frac{\beta}{6}(t_1^3 - L^3)\right\}\right]\right]\right\} \quad (11)$$

Our objective is to determine optimum value of t_1 to minimize TC. The values of t_1 for which

$\frac{\partial TC}{\partial t_1} = 0$ Satisfying the condition

$\left(\frac{\partial^2 TC}{\partial t_1^2}\right) > 0$

The optimal solution of the equation (11) is obtained by using Mathematica software. This has been illustrated by the following numerical example.

Numerical example

Crisp Model

We consider the following parametric values for $A = 300, a = 10, b = 5, c = 5, \ \beta = 0.05, \ L = 7, \ h = 4, \ S = 6$.

We obtain the optimal value of $t_1 = 1.06194$, $Q = 6407.77$ and minimum total cost $(TC) = 52822.7$.

Fuzzy Model

$\tilde{A} = (280, \ 300, \ 320)$, $\tilde{c} = (4, \ 5, \ 6)$, $\tilde{h} = (3, \ 4, \ 5)$ and $\tilde{S_c} = (7, \ 8, \ 9)$.

We obtain the optimal value of $t_1 = 1.0359$, $Q = 6417.29$ and minimum total cost $(\widetilde{TC}) = 39922.3$.

Conclusion

This paper presents fuzzy deterministic inventory model for time dependent deteriorating items that consider lead time as constant. This model provides retailers a mechanism to decide their economic order quantity and the cycle time for the items having stock dependent demand rate and non - zero lead time. The demand rate plays the most crucial role in the total inventory cost and plays an important role in deciding the optimal order quantity. The ordering cost, deterioration cost, holding cost and shortage cost are represented by triangular fuzzy number. Graded mean representation method is used for defuzzification. In this model the holding cost is constant and shortages are allowed during the lead time and completely backlogged. Thus, unless ordering cost is very high, in most of the cases the optimal economic order quantity is small with large number of cycles, which has proved by the numerical illustrations.

REFERENCES

Ameli, M., Mirzazadeh, A. and Shirazi, M. A., 2011. Economic order quantity model with imperfect items under fuzzy inflationary conditions. *Trends in Applied Sciences Research, 6*, 294–303.

Ben-daya, M. and Abdul Raouf, 1994. Inventory models inventory lead time as a decision variable, *Journal of Operational Research*, 45, 5, 579-582.

Chung, K. and Ting, P., A, 1993 heuristic for replenishment of deteriorating items with a linear trend in demand, *Journal of the Operational Research Society*, 1993, 44, 1235-1241.

Chang, S. C., Yao, J. S. and Lee, H. M., 1998. Economic reorder point for fuzzy backorder quantity. *European Journal of Operational Research, 109*, 183–202.

Chang, S. 1999. Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number. *Fuzzy Sets and Systems, 107*, 37–57.

Chang, H. C., Yao, J. S. and Ouyang, L. Y., 2006. Fuzzy mixture inventory model involving fuzzy random variable leadtime demand and fuzzy total demand. *European Journal of Operational Research, 169*, 65–80.

Covert, R.P and G. C. Philip, 1973, An EOQ model for items with Weibull distributed deterioration. *AIIE Transactions*, 5, 323-326.

Das, D. Effect of lead time on inventory: a static analysis, 1975, *Opl Res. Q*, 26, 273-282.

Foote. B, Kebriaei. N and Kumin. H, 1988, Heuristic policies for inventory ordering problems with long and randomly varying lead times. *Journal of Operations Management*, 7, 115-124.

Fujiwara, O. 1993, EOQ models for continuously deteriorating products using linear and exponential penalty costs, *European Journal of Operational Research*, 70, 104-14.

Gani, N. A. and Maheswari, S. 2010, Economic order quantity for items with imperfect quality where shortages are backordered in fuzzy environment. *Advances in Fuzzy Mathematics, 5*, 91–100.

Gen, M., Tsujimura, Y. and Zheng, D. 1997. An application of fuzzy set theory to inventory control models. *Computers and Industrial Engineering, 33*, 553–556.

Ghare and Schrader, 1963, A model for exponential decaying inventory. *Journal of Industrial Engineering*, 14(3), 238-43.

Kaufmann, A. and Gupta, M. M., 1991, *Introduction to Fuzzy arithmetic: Theory and Applications*. New York, NY: Van Nostrand Reinhold.

Kumar, S. and Rajput, U. S. 2015, Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. *Applied Mathematics, 6*, 496–509.

Kumar, N., and Kumar, S., 2016, Effect of learning and salvage worth on an inventory model for deteriorating items with inventory-dependent demand rate and partial backlogging with capability constraints. *Uncertain Supply Chain Management, 4*, 123–136.

Kumar, N. and Kumar, S. 2016a, Inventory model for Non- Instantaneous deteriorating items, stock dependent demand, partial backlogging, and inflation over a finite time horizon. *International journal of Supply and operations Management, 3*, 1168–1191.

Kumari, R., Kumar, N. and Singh, S. R. 2013, Two-warehouse inventory model of deteriorating items with three component demand rate and time-proportional backlogging rate in fuzzy environment. *International Journal of Industrial Engineering Computations, 4*, 587–598.

Lin, Y. J. 2008. A periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate. *Computers and Industrial Engineering, 54*, 666–676.

Magson, D. 1979, Stock control when the lead time cannot be considered constant, *Journal of the Operational Research Society,* 30, 317-322.

Maragatham, M. and Palani, R. 2016, An Inventory Model For Deteriorating Items With Lead Time And Time Dependent Holding Cost, *Aryabhatta Journal of Mathematics and Information (AJMI)*, 8 (2), ISSN (o) 2394-9309 (p) 0975-7139.

Naddor. E, 1966), Inventory Systems, *Wiley*, New York, 1966.

Nahmias, S. 1982. Perishable inventory theory a review. *Operations Research*, 30, 680-708.

Pal, S., Goswami, A. and Chaudhari, K.S. 1993. A deterministic inventory model for deteriorating items with stock dependent demand rate, *International Journal of Production Economics,* 32, 291-299.

Palani, R. and Maragatham, M. 2017. Fuzzy EOP model for controllable deterioration rate and time dependent demand and inventory holding cost, *International Journal of Current Research*, 05, 50295-50302.

Palani, R. and Maragatham, M. 2017a, Fuzzy EOQ model with shortages for products with controllable deterioration rate and time dependent demand and inventory holding cost, International Journal of Mathematical Archive, 8(8), 2017, 60-67

Park, K. 1987, Fuzzy-set theoretic interpretation of economic order quantity. *IEEE Transactions on Systems, Man, and Cybernetics,* 17, 1082–1084.

Rekha Rani Chaudhary, Vikas Sharma, Urvashi Chaudhary, 2013, ptimal Inventory Model for Time Dependent Decaying Items with Stock Dependent Demand Rate and Shortages, *International Journal of Computer Applications, (0975 – 8887), 79, 17.*

Roy, A. and Samanta, G. P. 2009, Fuzzy continuous review inventory model without backorder for deteriorating items. *Electronic Journal of Applied Statistical Analysis, 2*, 58–66.

Sadi-Nezhad, S., Memar Nahavandi, S. and Nazemi, J. 2011. Periodic and continuous inventory models in the presence of fuzzy costs. *International Journal of Industrial Engineering Computations, 2*, 167–178.

Tayal, S., Singh, S. R. and Sharma, R. 2014. An inventory model for deteriorating items with seasonal products and an option of an alternative market. *Uncertain Supply Chain Management, 3*, 69–86.

Vujosevic, M., Petrovic, D. and Petrovic, R. 1996, EOQ Formula when inventory cost is fuzzy International Journal Production. *Economics, 45*, 499–504.

Uthayakumar, R. and Valliathal, M. 2011, Fuzzy economic production quantity model for weibull deteriorating items with ramp type of demand. *International Journal of 100, Strategic Decision Sciences, 2*, 55–90.

Wee, H. M., Yu, J. and Chen, M. C. 2007, optimal inventory model for items with imperfect quality and shortage backordering. *International Journal of Management Science, 35*, 7–11.

Wilson R.H., 1934, A scientific routine for stock control. *Harvard Business Review*, 1934, 13, 116-128.

Yao, J. S. and Lee, H. M. 1996, Fuzzy inventory with backorder for fuzzy order quantity. *Information Sciences, 93*, 283–319.

Yao, J. S., Chang, S. C. and Su, J. S. 2000, Fuzzy inventory without backorder for fuzzy order quantity and fuzzy total demand quantity. *Computers & Operations Research, 27*, 935–962.

Yao, J. S. & Chiang, J. 2003, Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. *European Journal of Operational Research, 148*, 401–409.

Zadeh, L. A. 1965. Fuzzy sets. *Information and Control, 8*, 338–353.

Zimmermann, H. J. 1996. *Fuzzy Set Theory and Its Applications* (3rd ed.). Dordrecht: Kluwer Academic.

****