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INTEGRAL OPERATOR ON MEROMORPHIC p-VALENT FUNCTIONS WITH POSITIVE COEFFICIENTS

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ABSTRACT

In this paper we have introduced a family of meromorphic p- valent functions, by an integral operator and study some properties as coefficients inequalities, distortion theorems, closure theorems and radii of starlikeness and convexity.

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INTRODUCTION

A^*_p denote the class of functions f(z) of the form

$$f(z) = z^{-p} + \sum_{n=p}^{\infty} a_n z^n \ (a_n \geq 0, n \geq p, p \in N) \tag{1.1}$$

which are analytic and p-valent in the punctured unit disk $U^* = \{z \in C : 0 < |z| < 1\}$

Jum-Kim-Srivastava [1] defined an integral operator $I_p^\sigma f(z)$ for σ > 0 and for $f \in A_p^*$ as follows

$$I_p^\sigma f(z) = \frac{1}{z^{p+1}\Gamma(\sigma)} \int_0^z (log\frac{z}{t})^{\sigma-1} t^p f(z) dt \quad (n \epsilon N). \tag{1.2}$$

If f(z) is of the form (1.1), then

$$I_p^\sigma f(z) = z^{-p} + \sum_{n=p}^{\infty} (\frac{1}{n+p+1})^\sigma a_n z^n \ (n \geq p, p \in N). \tag{1.3}$$

In particular, when p=1, we have

$I^{\sigma}f(z) = z^{-1} + \sum_{n=1}^{\infty} (\frac{1}{n+2})^{\sigma} a_n z^n \ (n \geq p, p \in N)$

Let f and g be analytic in unit disk U, then g is said to be subordinate to f, written as $g \prec f$ or $g(z) \prec f(z)$, if there exists a Schwartz function ω, which is analytic in U with ω(0)=0 and $|\omega(z)| < 1(z \in U)$ such that g(z) = f(ω(z)). In particular, if the function f is univalent in U, we have the following equivalence ([2],[3])

$g(z) \prec f(z)(z \in U) \Leftrightarrow g(0) = f(0) and\ g(U) \subseteq f(U).$

Let $A_p^*(\sigma, b, A, B)$ denotes the class of functions of the form (1.1) which satisfies the condition

$$p - \frac{1}{b}\left\{\frac{z\left(I_p^{\sigma}f(z)\right)'}{I_p^{\sigma}f(z)} + p\right\} \prec p\frac{1+Az}{1+Bz} \tag{1.4}$$

Where $-1 \leq B < A \leq 1, p \in N, \sigma > 0, b$ non zero complex number.

We can re-write the condition (1.4) as

$$\left|\frac{z(I_p^{\sigma}f(z))' + pI_p^{\sigma}f(z)}{Bz(I_p^{\sigma}f(z))' + [Bp(1-b) + Abp\}I_p^{\sigma}f(z)}\right| < 1. \tag{1.5}$$

In this paper, coefficient inequalities, distortion theorem as well as closure theorem for the class $A_p^*(\sigma, b, A, B)$ are obtained.

2. COEFFICIENT INEQUALITIES:

Theorem 2.1: Let $f \in A_p^*$ be given by (1.1). Then $f \in A_p^*(\sigma, b, A, B)$ if and only if

$$\sum_{n=p}^{\infty}[(n+p)(1-B) - p|b|(A-B)]\left(\frac{1}{n+p+1}\right)^{\sigma} a_n \leq p|b|(A-B). \tag{2.1}$$

The result is sharp for the function f(z) given by

$$f(z) = z^{-p} + \left(\frac{p|b|(A-B)}{[(n+p)(1-B) - p|b|(A-B)]}\right)(n+p+1)^{\sigma} z^k, (k \geq p, n \in N). \tag{2.2}$$

Proof: Assuming that the inequality (2.1) holds true then from (2.1),we find that

$$\left|\frac{z\left(I_p^{\sigma}f(z)\right)' + pI_p^{\sigma}f(z)}{Bz\left(I_p^{\sigma}f(z)\right)' + [Bp(1-b) + Abp\}I_p^{\sigma}f(z)}\right| \leq \frac{\sum_{n=p}^{\infty} (n+p)\ (\frac{1}{n+p+1})^{\sigma} a_n}{p|b|(A-B) + \sum_{n=p}^{\infty}[B(n+p) + p|b|(A-B)](\frac{1}{n+p+1})^{\sigma} a_n} < 1.$$

$(z \in U^*, z \in C, |z| < 1).$

Hence, by the Maximum Modulus Theorem we have $f(z) \in A_p^*(\sigma, b, A, B)$.

Conversely, suppose that $f(z) \in A_p^*(\sigma, b, A, B)$.Then from (1.5) we have

$$\left|\frac{z\left(I_p^{\sigma}f(z)\right)' + pI_p^{\sigma}f(z)}{Bz\left(I_p^{\sigma}f(z)\right)' + [Bp(1-b) + Abp\}I_p^{\sigma}f(z)}\right| = \left|\frac{\sum_{n=p}^{\infty}(n+p)\left(\llbracket\frac{1}{n+p+1}\right)\rrbracket^{\sigma} a_n\ z^n}{p|b|(A-B) + \sum_{n=p}^{\infty}[B(n+p) + p|b|(A-B)]\left(\llbracket\frac{1}{n+p+1}\right)\rrbracket^{\sigma} a_n z^n}\right| < 1$$

If we choose z to be real and $z \to 1^-$, we get

$$\sum_{n=p}^{\infty}[(n+p)(1-B) - p|b|(A-B)]\left(\frac{1}{n+p+1}\right)^{\sigma} a_n \leq p|b|(A-B).$$

which give (2.1).

3. DISTORTION THEOREM:

Theorem 3.1: If the function f(z) defined by (1.1) is in the class $A_p^*(\sigma, b, A, B)$. Then for $0 < |z| = r < 1$, we have

$$r^{-p} - \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) r^p \le |f(z)| \le r^{-p} + \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) r^p \tag{3.1}$$

where equality holds true for the function

$$f(z) = z^{-p} + \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) z^p \tag{3.2}$$

Proof: Since $f(z) \in A_p^*(\sigma, b, A, B)$, then from (2.1)

$$[2p(1-B) - p|b|(A-B)]\left(\frac{1}{2p+1}\right)^{\sigma} \sum_{n=p}^{\infty} |a_n| \quad \le \sum_{n=p}^{\infty} [(n+p)(1-B) - p|b|(A-B)]\left(\frac{1}{n+p+1}\right)^{\sigma} a_n$$
$$\le p|b|(A-B).$$

we conclude that

$$\sum_{n=p}^{\infty} |a_n| \le \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) \tag{3.3}$$

Thus for $0 < |z| = r < 1$,

$$|f(z)| \le |z|^{-p} + \sum_{n=p}^{\infty} |a_n| z^n \le r^{-p} + r^p \sum_{n=p}^{\infty} |a_n|$$

or

$$|f(z)| \le r^{-p} + \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) r^p \tag{3.4}$$

and

$$|f(z)| \ge |z|^{-p} - \sum_{n=p}^{\infty} |a_n| z^n \ge r^{-p} - r^p \sum_{n=p}^{\infty} |a_n|$$

or

$$|f(z)| \ge r^{-p} - \left(\frac{p|b|(A-B)(2p+1)^{\sigma}}{[2p(1-B)-p|b|(A-B)]}\right) r^p \tag{3.5}$$

On using (3.4) and (3.5) inequality (3.1) follows.

4. CLOSURE THEOREM

Theorem 4.1: Let

$$f_{p-1}(z) = z^{-p} \ and \ f_n(z) = z^{-p} + \left(\frac{p|b|(A-B)(n+p+1)^{\sigma}}{[(n+p)(1-B)-p|b|(A-B)]}\right) z^n \tag{4.1}$$

$for \ n \ge p$, then $f(z) \in A_p^*(\sigma, b, A, B)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=p-1}^{\infty} \mu_n f_n(z), \text{ where } \mu_n \ge 0 \text{ and } \sum_{n=p-1}^{\infty} \mu_n = 1. \tag{4.2}$$

Proof: Let f(z) can be expressed in the form (4.1), then

$$f(z) = \sum_{n=p-1}^{\infty} \mu_n f_n(z) \ = z^{-p} + \sum_{n=p}^{\infty} \left(\frac{p|b|(A-B)(n+p+1)^{\sigma}\mu_n}{[(n+p)(1-B)-p|b|(A-B)]}\right) z^n .$$

Then,

$$\sum_{n=p}^{\infty} \frac{p|b|(A-B)(n+p+1)^{\sigma}\mu_n}{[(n+p)(1-B)-p|b|(A-B)]} [(n+p)(1-B) - p|b|(A-B)]\left(\frac{1}{n+p+1}\right)^{\sigma}$$

$$= \sum_{n=p}^{\infty} p|b|(A-B)\mu_n = p|b|(A-B) \sum_{n=p}^{\infty} \mu_n \le p|b|(A-B).$$

So, from (2.1), it follows that $f(z) \in A_p^*(\sigma, b, A, B)$.

Conversely, let $f(z) \in A_p^*(\sigma, b, A, B)$. From theorem 2.1, we have

$$a_n \le \frac{p|b|(A-B)(n+p+1)^{\sigma}}{[(n+p)(1-B)-p|b|(A-B)]} \ for \ n \ge p.$$

Setting

$\mu_n = \frac{(n+p)(1-B)-p|b|(A-B)}{p|b|(A-B)}(\frac{1}{n+p+1})^\sigma \; for\; n \geq p.$

And $\mu_{p-1} = \sum_{n=p}^{\infty} \mu_n$

It follows that

$$f(z) = \sum_{n=p-1}^{\infty} \mu_n f_n(z).$$

This completes the proof.

5. RADII OF STARLIKENESS AND CONVEXITY:

Theorem 5.1: Let the function f(z) defined by (1.1) be in the class $A_p^*(\sigma, b, A, B)$. Then

(i) f is meromorphically p-valent starlike of order δ $(0 \leq \delta \leq p)$ in the disk $|z| < r_1$,where

$$r_1 = r_1(p,\sigma,b,A,B) = \min_{n\geq p}[\frac{(n+p)(1-B)-p|b|(A-B)}{p|b|(A-B)}\left(\frac{1}{n+p+1}\right)^\sigma \frac{p-\delta}{n+\delta}]^{\frac{1}{n}}. \tag{5.1}$$

(ii) f is meromorphically p-valent convex of order δ $(0 \leq \delta \leq p)$ in the disk $|z| < r_2$,where

$$r_2 = r_2(p,\sigma,b,A,B) = \min_{n\geq p}[\frac{(n+p)(1-B)-p|b|(A-B)}{p|b|(A-B)}\left(\frac{1}{n+p+1}\right)^\sigma \frac{p(p-\delta)}{n(n+\delta)}]^{\frac{1}{n}}. \tag{5.2}$$

Proof: (i) Using definition (1.1), we observe that

$$\left|\frac{z f'(z)+pf(z)}{z f'(z)+(2\delta-p)f(z)}\right| \leq \frac{\sum_{n=p}^{\infty}(n+p)|a_n||z|^n}{2(p-\delta)-\sum_{n=p}^{\infty}(n-p+2\delta)|a_n||z|^n} \leq 1, (|z| < r_1; 0 \leq \delta < 1). \tag{5.3}$$

This last inequality (5.3) holds true if

$$\sum_{n=p}^{\infty} (\frac{n+\delta}{p-\delta}) |a_n||z|^n \leq 1.$$

In view of (2.1), the last inequality is true if

$$(\frac{n+\delta}{p-\delta})|z|^n \leq [\frac{(n+p)(1-B)-p|b|(A-B)}{p|b|(A-B)}\left(\frac{1}{n+p+1}\right)^\sigma]\; (n \geq p, p \in N).$$

which on solving gives (5.1).

(ii) Using definition (1.1), we observe that

$$\left|\frac{z f''(z)+(1+p)f'(z)}{z f''(z)+(1+2\delta-p)f'(z)}\right| \leq \frac{\sum_{n=p}^{\infty} n(n+p)|a_n||z|^n}{2p(p-\delta)-\sum_{n=p}^{\infty} n(n-p+2\delta)|a_n||z|^n} \leq 1, (|z| < r_2; 0 \leq \delta < 1). \tag{5.4}$$

This last inequality (5.4) holds true if

$$\sum_{n=p}^{\infty} (\frac{n}{p}\frac{(n+\delta)}{(p-\delta)}) |a_n||z|^n \leq 1.$$

In view of (2.1), the last inequality is true if

$$(\frac{n}{p}\frac{(n+\delta)}{(p-\delta)})|z|^n \leq [\frac{(n+p)(1-B)-p|b|(A-B)}{p|b|(A-B)}\left(\frac{1}{n+p+1}\right)^\sigma]\; (n \geq p, p \in N).$$

which on solving gives (5.2).

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